NEUMANN AND INTERMEDIATE BIHARMONIC EIGENVALUE PROBLEMS ON SINGULARLY PERTURBED DOMAINS

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Domain perturbation theory for the eigenvalues of the Laplace operator on families of bounded, Lipschitz domains of $\mathbb{R}^N$ is nowadays a well-understood, yet complicated subject. For the biharmonic operator, the situation is more involved, mainly due to two additional hurdles: 1) intermediate and Neumann boundary conditions are very sensitive to the variation of the curvature of the boundary; 2) standard techniques, such as the separation of variables, are not available.
After a review of the main results and counterexamples for the Laplace operator and the biharmonic operator, I will focus on three specific singular perturbations where spectral continuity fails: the dumbbell domain (Neumann b.c); a Lipschitz domain, whose boundary is locally defined as the graph of a fast oscillating smooth function (Intermediate b.c.); thin annuli in $\mathbb{R}^2$ (Neumann b.c).
A particularly striking result is the following. Let Ω be a bounded domain in $\mathbb{R}^2$ with smooth boundary $\partial\Omega$, and let ω_h be the set of points in Ω whose distance from the boundary is smaller than h. Then the eigenvalues of the biharmonic operator on ω_h with Neumann boundary conditions do not converge to the eigenvalues of the biharmonic operator on $\partial\Omega$; in fact, they converge to the eigenvalues of a system of differential equations on $\partial\Omega$.
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