The multiplicity of λ_1 in genus 3

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February 14, 2022

The maximum multiplicity of the first positive eigenvalue λ_1 of the Laplacian on Riemannian surfaces has been studied by several authors. Its value is known for surfaces of small complexity, but its asymptotic rate of growth in terms of the complexity is unknown. In this talk, I will discuss joint work with Bram Petri in which we prove that the multiplicity of λ_1 is at most 8 for closed hyperbolic surfaces of genus 3 and that this bound is achieved by the most symmetric such surface, the Klein quartic. The proof of the upper bound splits into two parts depending on the value of λ_1. When λ_1 is not too large, we combine ideas of Sévennec for bounding the multiplicity on arbitrary surfaces with the Faber-Krahn inequality. When λ_1 is relatively large, we prove bounds using the Selberg trace formula and rigorous numerical calculations, a strategy adapted from the theory of sphere packings with many potential future applications. As for the lower bound on the multiplicity at the Klein quartic, it uses both trace formula techniques and representation theory.