Growth and divisor of complexified horocycle eigenfunctions

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Furstenberg Theorem on unique ergodicity of horocycle flow over compact hyperbolic surfaces can be passed through a semiclassical quantization. We then arrive to a plenty of *horocycle eigenfunctions* u defined at the hyperbolic plane $\mathbb{C}^+$. They enjoy

$$\left(-y^2\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)+2i\tau y\frac{\partial}{\partial x}\right)u(x+iy)=s^2u(x+iy),\quad x+iy\in\mathbb{C}^+,$$

with $\tau\to\infty$, $s=o(\tau)$, $s,\tau\in\mathbb{R}$, and possess Quantum Unique Ergodicity ($\hbar=1/\tau$). At the left-hand side, we recognize *magnetic* Hamiltonian at hyperbolic plane.

Such functions can be analytically continued to a neighborhood of $\mathbb{C}^+$ in its complexification. The latter is just $\{(X,Y)\colon X,Y\in\mathbb{C}\}$. We establish asymptotic estimates for the growth of these continuations as $\tau\to\infty$, and for de Rham currents given by their divisors.

The main feature making this setting different from that of free quantum particle (and geodesic flow) is presence of *gauge factors* in calculations and in answers.