Variations on the Weyl law for the Steklov problem on surfaces

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Weyl's law describes the distribution of eigenvalues as the spectral parameter goes to infinity. For surfaces with smooth boundary, a superpolynomially precise Weyl law for the Steklov problem for the Laplacian can be obtained via the theory of pseudodifferential operators on circles. Adapting those methods, we obtain a complete asymptotic expansion (as well as spectral invariants) for the eigenvalues of the Steklov problem associated with a Schrödinger operator*, and a remainder estimate depending on the regularity when the boundary is not smooth but merely of class C^r.**

When the boundary is merely Lipschitz, it is a well-known open problem to prove that the eigenvalues still obey a Weyl law. Following the works of Suslina, and later Agranovich, we know that piecewise C^1 is sufficient to obtain such asymptotics. Using conformal techniques and through a perturbation of the pseudodifferential results we can obtain a Weyl law for the Steklov problem not only for domains with Lipschitz boundary, but also in more singular settings that include every polynomial cusp that is slow enough to obtain a discrete Steklov spectrum.***

In this talk, I will describe the Weyl laws that we have obtained over the last few years, briefly mention the ideas behind the proofs and, time permitting, talk about the directions we could go from there.

* Joint work with Simon St-Amant (Cambridge)
** Joint work with Broderick Causley (McGill)
*** Joint work with Mikhail Karpukhin (Caltech) and Iosif Polterovich (Montréal)