Spectral asymptotics of noisy non-selfadjoint operators

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The spectral theory of non-selfadjoint operators is an old and highly developed subject. Yet it still poses many new challenges crucial for the understanding of modern problems such as scattering systems, open or damped quantum systems, the analysis of the stability of solutions to non-linear PDEs, and many more. The lack of powerful tools readily available for their selfadjoint counterparts, such a general spectral theorem or variational methods, makes the analysis of the spectra of non-selfadjoint operators a subtle and highly varied subject. One fundamental issue of non-selfadjoint operators is their intrinsic sensitivity to perturbations, indeed even small perturbations can change the spectrum dramatically. This spectral instability, also called pseudospectral effect, was initially considered a drawback as it can be at the origin of severe numerical errors. However, recent works in semiclassical analysis and random matrix theory have shown that this pseudospectral effect also leads to new and beautiful results concerning the spectral distribution and eigenvector localization of non-selfadjoint operators with small random perturbations. In this talk, I will discuss recent results and some fundamental techniques involved in the analysis. The talk is partly based on joint work with Anirban Basak, Stéphane Nonnenmacher, Johannes Sjöstrand and Ofer Zeitouni