The maximal spectral gap of a hyperbolic surface

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A hyperbolic surface is a surface with metric of constant curvature -1. The spectral gap between the first two eigenvalues of the Laplacian on a closed hyperbolic surface contains a good deal of information about the surface, including its connectivity, dynamical properties of its geodesic flow, and error terms in geodesic counting problems. For arithmetic hyperbolic surfaces the spectral gap is also the subject of one of the biggest open problems in automorphic forms: Selberg's eigenvalue conjecture.

It was an open problem from the 1970s whether there exist a sequence of closed hyperbolic surfaces with genera tending to infinity and spectral gap tending to 1/4. (The value 1/4 here is the asymptotically optimal one.) Recently we proved that this is indeed possible. I'll discuss the very interesting background of this problem in detail as well as some ideas of the proof.

This is joint work with Will Hide.