Łojasiewicz-type inequalities for the H-functional near simple bubble trees

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The H-functional E is a natural variant of the Dirichlet energy along maps u from a closed surface S into $\mathbb{R}^3$. Critical points of E include conformal parameterisations of constant mean curvature surfaces in $\mathbb{R}^3$. The functional itself is unbounded from above and below on $H^1(S, \mathbb{R}^3)$, but all critical points have H-energy E at least $4\pi/3$, with equality attained if and only if we are parametrising a round sphere (so S itself must be a sphere) - this is the classical isoperimetric inequality.

Here we will address the simple question: can one approach the natural lower energy bound by critical points along fixed surfaces of higher genus? In fact we prove more subtle quantitative estimates for any (almost-)critical point whose energy is close to $4\pi/3$. Standard theory tells us that a sequence of (almost-)critical points on a fixed torus T, whose energy approaches $4\pi/3$, must bubble-converge to a sphere: there is a shrinking disc on the torus that gets mapped to a larger and larger region of the round sphere, and away from the disc our maps converge to a constant. Thus the limiting object is really a map from a sphere to $\mathbb{R}^3$, and the challenge is to compare maps from a torus with the limiting map (i.e. a change of topology in the limit). In particular we can prove a gap theorem for the lowest energy level on a fixed surface and estimate the rates at which bubbling maps u are becoming spherical in terms of the size of $dE[u]$ - these are commonly referred to as Łojasiewicz-type estimates.

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