A new approach to the hot spots conjecture

Jonathan Rohleder

Stockholm University

November 1, 2021

It is a conjecture going back to J. Rauch (1974) that the hottest and coldest spots in an insulated homogeneous medium such as an insulated plate of metal should converge to the boundary, for "most" initial heat distributions, as time tends to infinity. This so-called hot spots conjecture can be phrased alternatively as follows: the eigenfunction(s) corresponding to the first non-zero eigenvalue of the Neumann Laplacian on a Euclidean domain should take its maximum and minimum on the boundary only. This has been proven to be false for certain domains with holes, but it was shown to hold for several classes of simply connected or convex planar domains. One of the most recent advances is the proof for all triangles given by Judge and Mondal (Annals of Math. 2020). The conjecture remains open in general for simply connected or at least convex domains. In this talk we provide a new approach to the conjecture. It is based on a non-standard variational principle for the eigenvalues of the Neumann and Dirichlet Laplacians.