Semiclassical analysis and the convergence of the finite element method

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An important problem in numerical analysis is the solution of the Helmholtz equation in exterior domains, in variable media; this models the scattering of time-harmonic waves. The Finite Element Method (FEM) is a flexible and powerful tool for obtaining numerical solutions, but difficulties are known to arise in obtaining convergence estimates for FEM that are uniform as the frequency of waves tends to infinity. I will describe some recent joint work with David Lafontaine and Euan Spence that yields new convergence results for the FEM which are uniform in the frequency parameter. The essential new tools come from semiclassical microlocal analysis and the use of the functional calculus. Another ingredient is a slightly surprising new resolvent estimate.