Nodal deficiency via equipartitions and Dirichlet-to-Neumann maps

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Courant's nodal domain theorem, which says the nth Laplacian eigenfunction has at most n nodal domains, is almost always a strict inequality. The extent to which it fails to be sharp is measured by the nodal deficiency. Despite much study, this quantity is still not very well understood except in highly symmetric cases. However, in the last decade two general formulas for the nodal deficiency were established.

The first was given in 2012 by Berkolaiko, Kuchment and Smilansky, using an energy functional defined on the space of equipartitions of the domain. More recently, with Jones and Marzuola, I obtained a formula for the nodal deficiency in terms of a two-sided Dirichlet-to-Neumann map defined on the nodal set.

After reviewing both of these results, I will describe new work (with Gregory Berkolaiko, Yaiza Canzani and Jeremy Marzuola) that demonstrates a direct connection between these seemingly different approaches to nodal deficiency. Among other things, it gives a method for using the Dirichlet-to-Neumann map to calculate eigenfunctions for the Hessian of the equipartition energy, and provides insight into the theory of spectral minimal partitions.