Eigenvalue estimates and asymptotics for weighted pseudodifferential operators with singular measures in the critical case

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March 1, 2021

We consider self-adjoint operators of the form $\mathbf{T}_{P,\mathfrak{A}} = \mathfrak{A}^* P \mathfrak{A}$ in a domain $\Omega \subset \mathbb{R}^{\mathbf{N}}$, where $\mathfrak{A}$ is an order $-l = -\mathbf{N}/2$ pseudodifferential operator in Ω and P is a signed Borel measure with compact support in Ω. Measure P may contain singular component. For a wide class of measures we establish eigenvalue estimates for operator $\mathbf{T}_{P,\mathfrak{A}}$. In case of measure P being absolutely continuous with respect to the Hausdorff measure on a Lipschitz surface of an arbitrary dimension, we find the eigenvalue asymptotics. The order of eigenvalue estimates and asymptotics does not depend on dimensional characteristics of the measure, in particular, on the dimension of the surface supporting the measure. The typical example is the spectral problem $\lambda(1 - \Delta)^{N/2}u = Pu$.