Free boundary minimal surfaces with connected boundary and arbitrary genus

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Besides their self-evident geometric significance, which can be traced back at least to Courant, free boundary minimal surfaces also naturally arise in partitioning problems for convex bodies, in capillarity problems for fluids and, as has significantly emerged in recent years thanks to work of Fraser and Schoen, in connection to extremal metrics for Steklov eigenvalues for manifolds with boundary (i. e. for eigenvalues of the corresponding Dirichlet-to-Neumann map).

The theory has been developed in various interesting directions, yet many fundamental questions remain open. One of the most basic ones can be phrased as follows: does the Euclidean unit ball contain free boundary minimal surfaces of any given topological type? In spite of significant advances, the answer to such a question has proven to be very elusive. I will present some joint work with Giada Franz and Mario Schulz where we answer (in the affirmative) the well-known question whether there exist in B^3 (embedded) free boundary minimal surfaces of genus one and one boundary component. In fact, we prove a more general result: for any g there exists in B^3 an embedded free boundary minimal surface of genus g and connected boundary. The proof builds on global variational methods, in particular on a suitable equivariant counterpart of the Almgren-Pitts min-max theory, and on a striking application of Simon's lifting lemma.