
SMALL DENOMINATORS AND LARGE NUMERATORS OF QUASIPERIODIC SCHRÖDINGER OPERATORS

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We initiate an approach to simultaneously treat numerators and denominators of Green's functions arising from quasi-periodic Schrödinger operators, which in particular allows us to study completely resonant phases of the almost Mathieu operator.

Let $(H_{\lambda,\alpha,\theta}u)(n) = u(n+1) + u(n-1) + 2\lambda \cos 2\pi(\theta + n\alpha)u(n)$ be the almost Mathieu operator on $\ell^2(\mathbb{Z})$, where $\lambda, \alpha, \theta \in \mathbb{R}$. Let

$$\beta(\alpha) = \limsup_{k \rightarrow \infty} -\frac{\ln \|k\alpha\|_{\mathbb{R}/\mathbb{Z}}}{|k|}.$$

We prove that for any θ with $2\theta \in \alpha\mathbb{Z} + \mathbb{Z}$, $H_{\lambda,\alpha,\theta}$ satisfies Anderson localization if $|\lambda| > e^{2\beta(\alpha)}$. This confirms a conjecture of Avila and Jitomirskaya [The Ten Martini Problem. *Ann. of Math. (2)* 170 (2009), no. 1, 303–342] and a particular case of a conjecture of Jitomirskaya [Almost everything about the almost Mathieu operator. II. XIth International Congress of Mathematical Physics (Paris, 1994), 373–382, Int. Press, Cambridge, MA, 1995].