Two balls maximize the third Neumann eigenvalue in hyperbolic space

Richard Laugesen

Department of Mathematics, University of Illinois, USA

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The second eigenvalue of the Neumann Laplacian is maximal for a ball under volume normalization, by Weinberger (1956). For simply connected planar domains, Szegö did it two years earlier using conformal mapping. The third eigenvalue is maximal for a disjoint union of two balls having the same size, by Bucur and Henrot (2019). For simply connected planar domains, Girouard, Nadirashvili and Polterovich did it ten years earlier. This talk explores the general approach of these authors in terms of trial functions constructed by even reflection ("folding over a hyperplane"), and provides a new proof of a key step in Bucur and Henrot's argument. A starring role is played by a homotopy result of Petrides that has the flavor of the Borsuk-Ulam theorem. This new approach yields a two-ball upper bound on the third eigenvalue for domains in hyperbolic space. (Joint work with Pedro Freitas.)