Nodal Sets of Eigenfunctions: Progress via Optimal Transport

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An old and important question about Laplacian eigenfunctions is to understand the size of the set where they vanish (their nodal set): the idea is that eigenfunctions corresponding to larger frequencies should oscillate more and thus vanish on a larger set. The same principle should then also be true for linear combinations of high-frequency eigenfunctions (in one dimension, this follows from Sturm-Liouville theory). Recent progress on this question is based on the notion of optimal transport and a very simple idea which we formalize: if it's easy to buy milk, then there must be many supermarkets (and, conversely, if there are only few supermarkets some people have to travel a large distance to buy milk). This turns into a geometric inequality that is interesting in its own right. The methods are self-contained and elementary but many open problems remain.