Index of minimal surfaces in spheres and eigenvalues of the Laplacian

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The Laplacian is a canonical second order elliptic operator defined on any Riemannian manifold. The study of upper bounds for its eigenvalues is a classical problem of spectral geometry going back to J. Hersch, P. Li and S.-T. Yau. It turns out that the optimal isoperimetric inequalities for Laplacian eigenvalues are closely related to minimal surfaces in spheres. At the same time, the index of a minimal surface is defined as a number of negative eigenvalues of a different second order elliptic operator. It measures the instability of the surface as a critical point of the area functional.

In the present talk we will discuss the interplay between index and Laplacian eigenvalues, and present some recent applications, including a new bound on the index of minimal spheres as well as the optimal isoperimetric inequality for Laplacian eigenvalues on the projective plane.