A Laplace operator with boundary conditions singular at one point

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In this talk I will present some work with Rozenblum from 2009. While it has been known for more than half a century that the Laplace operator on a smooth, bounded domain may have essential spectrum if the boundary conditions are suitably chosen, typical choices involved non-local operators. In this talk I will show, with very elementary arguments, that even local boundary conditions, singular even just at a single point - can have a huge impact on the spectrum and eigenfunctions. The example we consider, first proposed by Berry and Dennis. still has empty essential spectrum and compact resolvent. However Weyl's law fails completely because the spectrum becomes unbounded below. The positive eigenvalues still obey Weyl asymptotics, to leading order; however the (absolute values of the) negative eigenvalues do not obey a power law distribution.

I will also make some remarks and ask some questions about nodal domains, which were not addressed in our paper.