Chantal David, Concordia University.
Non-vanishing cubic Dirichlet L-functions at $s = 1/2$
Joint work with A. Florea and M. Lalin.

A famous conjecture of Chowla predicts that $L(1/2, \chi) \neq 0$ for Dirichlet L-functions attached to primitive characters χ. It was conjectured first in the case where χ is a quadratic character, which is the most studied case. For quadratic Dirichlet L-functions, Soundararajan then proved that at least 87.5% of the quadratic Dirichlet L-functions do not vanish at $s = 1/2$, by computing the first two mollified moments. Under GRH, there are slightly stronger results by Ozlek and Snyder obtained by computing the one-level density.

We consider in this talk cubic Dirichlet L-functions. There are few papers in literature about Dirichlet cubic L-functions, compared to the abundance of papers on Dirichlet quadratic L-functions, as this family is more difficult, in part because of the cubic Gauss sums. The first moment for $L(1/2, \chi)$ where χ is a primitive cubic character was computed by Baier and Young over $\mathbb{Q}$ (the non-Kummer case), by Luo over $\mathbb{Q}(\sqrt{-3})$ (the Kummer case), and by David, Florea and Lalin over function fields, in both the Kummer and non-Kummer case. Bounding the second moment, those authors could obtain lower bounds for the number of non-vanishing cubic twists, but not a positive proportion. Moreover, for the case of Dirichlet cubic L-functions, computing the one-level density under the GRH also gives lower bounds which are weaker than any positive proportion.

We prove in this talk that there is a positive proportion of cubic Dirichlet L-functions non-vanishing at $s = 1/2$ over function fields. This can be achieved by using the recent breakthrough work on sharp upper bounds for moments of Soundararajan and Harper. There is nothing special about function fields in our proof, and our results would transfer over number fields (but we would need to assume GRH in this case).