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Complexity of MULTIPACKING in graphs

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A *multipacking* in an undirected graph G is a set M of vertices such that for every positive integer d and for every vertex v of G, there are at most d vertices of M within distance at most d of v. This notion was introduced by Brewster, Mynhardt and Teshima, see [3, 7]. Note that any multipacking must be, in particular, a distance 3-independent set.

This definition is originally motivated by the dual notion (in the sense of Integer Linear Programming) of *broadcast domination* in graphs. Finding the optimal solution to the broadcast domination problem can be done, rather surprisingly, in polynomial time [5].

The formal associated decision problem is:

MULTIPACKING
Input: A graph G, an integer k.
Question: Does G admit a multipacking of size at least k?

MULTIPACKING is clearly in NP. The open problem is as follows:

Problem: Is MULTIPACKING NP-complete? Is it polynomial-time solvable?

Note that MULTIPACKING can be solved in linear time on trees [2, 6, 7] and in cubic time on strongly chordal graphs [2]. It can be approximated up to a constant factor [1]. The variant where the packing constraint is only required for distances that are at most a fixed value, is NP-complete [9]. The generalization to directed graphs is also NP-complete [4].

This problem was communicated to me in 2016 by R. C. Brewster. It was also stated in [7, 8] and has been mentioned in [1, 2, 4].

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