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# Constructing sort of $\Delta$-regular graphs to challenge the IRUP property for bin packing

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We study pairs $(G, w)$ where $G = (V, E)$ is a simple graph and $w : E \mapsto \mathbf{N}$ associates a natural to each edge. Fixed a natural $\Delta$, a subset $F$ of $E$ is called *light* if $w(F) \leq \Delta$ and *tight* when $w(F) = \Delta$, where $w(F) := \sum_{e \in F} w(e)$. Denote by $\delta(v)$ the *star of* $v$, i.e., the set of edges incident to node $v$. When $|V| = 2n$ is an even natural, and each star is tight, then $(G, w)$ is called a $\Delta$*-reg*. Notice that $\{\delta(v) : v \in V\}$ is a family of $2n$ tight sets such that every edge $e \in E$ belongs to precisely two of them. This implies that we need to take at least $n$ light sets if we want every edge $e$ of $G$ to belong to at least one of them. A $\Delta$-reg is said to be *of gap* $k$ if at least $n + k$ light sets are required in order to cover $E$. Can you construct a $\Delta$-reg of gap $k > 1$? If yes, then you have obtained 2-gappers for the IRUP property for the bin packing problem, which currently are not known to exist yet. Constructing a family of $\Delta$-regs of arbitrarily high gap would be even bigger.

Alberto Caprara taught us the above approach to construct $k$-gappers for the IRUP. This approach is not complete: it might be doomed to fail even if $k$-gappers actually exist, but it has the advantage to cast the problem as a purely combinatorial problem expressed in terms of graphs, where some extra intuition could possibly help. Indeed, we have this vague impression that $\Delta$-regs of gap bigger than one should exist.

It has been experimentally assessed that the gap between the optimal values of bin packing and fractional bin packing, if the latter is rounded up to the closest integer, is almost always null. Prior to [1], known counterexamples to this for integer input values involved fairly large numbers. Specifically, the first 1-gapper for the IRUP property was derived in 1986 and involved a bin capacity of the order of a billion. Later in 1998 a counterexample with a bin capacity of the order of a million was found. In [1], a large number of small (some with $n = 6$, the smallest possible) $\Delta$-regs of gap 1 were built, leading to an even larger number of 1-gapper for the IRUP property with bin capacity of the order of a hundred, showing that the gap may be positive even for numbers which arise in customary applications. These $\Delta$-regs are constructed starting from the Petersen graph and using the fact that it is fractionally, but not integrally, 3-edge colorable.

In recent unpublished work I have somewhat characterized the cone of where this sort of 1-gappers may lay, but again: is it possible to push the gap above 1?

## References

[1] A. Caprara, M. Dell'Amico, J.C. Díaz Díaz, M. Iori, R. Rizzi, Friendly Bin Packing Instances without Integer Round-up Property, *Mathematical Programming Series A and B* 150(1):5–17, 2015.