

## The maximum edge $q$ -coloring problem

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We consider the problem of coloring edges of a graph subject to the following constraints: for every vertex  $v$ , all the edges incident with  $v$  have to be colored with at most  $q$  colors. The goal is to find a coloring satisfying the above constraints and using the maximum number of colors. Notice that the notion of coloring is different than in the classical edge coloring problem, as neighboring edges can have the same color.

The problem is NP-hard, and moreover, hard to approximate within a factor of  $3/2$  assuming the Unique Games Conjecture [2, 3]. On the other hand, the best approximation algorithm achieves a factor of 2 [1]. Therefore, the main open problem is to close the gap between these approximation and hardness results. We would like to emphasize that the hardness result from [3] holds even on bipartite graphs. For a detailed presentation of the previous work we refer to [2].

Recently, a related problem, named min-max edge  $q$ -coloring, was introduced and shown to be NP-hard, by Larjomaa and Popa [4]. In this related problem the goal is to find an edge  $q$ -coloring such that the size of the largest color class is minimized. There are no known approximation algorithms on general graphs for the min-max edge  $q$  coloring problem, and therefore, designing such algorithms is an interesting open problem.

## References

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