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The Generalized Inverse Voronoi Problem

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Any planar straight-line graph (PSLG) subdivides the plane into cells, some of which may be unbounded. The *Voronoi diagram* (also commonly referred to as *Dirichlet tesselation*, or *Thiessen polygon*) of a set S of n points (also called *sites* or *generators*) is a PSLG with n cells, where each cell belongs to one generator from S and consists of all points in the plane that are closer to that generator than to any other in S.

Let G be a given PSLG, whose cells can be considered bounded and convex for all practical purposes. The *Inverse Voronoi Problem* (IVP) consists of deciding whether G coincides with the Voronoi diagram of some set S of points in the plane, and if so, finding S. This problem is studied in [2, 3, 5, 6].

In the IVP, the set S is limited to have one point per cell; a generalized version of this problem (GIVP) allows more than one point per cell. In this case, new vertices and edges may be added to G, but the original ones must be kept. With this relaxation the set S always exists, hence we are interested in minimizing its size. An obvious lower bound is the number of cells or faces of G, which can only be achieved if G is a Voronoi diagram.

An algorithm for solving GIVP in $\mathbb{R}^2$ is presented in [1, 7], which generates $\mathcal{O}(E)$ sites in the worst case, where E is the number of edges of G (provided that the smallest angle of G is constant). This bound is asymptotically optimal for tesselations with such angular constraints.

Banerjeee et al. describe another algorithm for GIVP in $\mathbb{R}^2$ [4], which generates $\mathcal{O}(V^3)$ sites, where V is the size of a refinement of G such that all faces are triangles with acute angles. Given an arbitrary PSLG, there does not appear to be any known polynomial upper bound on the size of its associated acute triangulation. Moroever, a comparison with [1, 7] is not straightforward.

In general, we are interested in finding tighter (upper and lower) bounds for the number of generators, than those given by [1, 7] and [4].

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