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The $(2,1)$-C1P

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A binary matrix M has the *consecutive-ones property* (C1P) if its columns can be permuted such that for each row, the set of columns containing 1's entries is consective in this permutation. It can be decided in polynomial-time if M has the C1P [Tucker, 1972].

We consider the slight relaxation of this property, namely the (k, δ)-C1P. A binary matrix M has the (k, δ)-C1P for integers k, δ if the columns of M can be permuted such that each row contains at most k *blocks* (consecutive runs) of 1's entries, while no two neighboring blocks of 1's are separated by a gap of more than δ 0 entries. Note that the (1,0)-C1P is the classical C1P. To our surpise, for every $k \geq 2, \delta \geq 1, (k, \delta) \neq (2,1)$, deciding the (k, δ)-C1P for M is NP-hard, and for every $\delta \geq 1$, deciding the (∞, δ)-C1P (number of blocks is unbounded) is NP-hard [Maňuch et al., 2011]. It was shown in [Goldberg et al., 1995] that deciding the (k, ∞)-C1P (under the name k-C1P: number of gaps is unbounded) for M is NP-hard.

This means that the curious case of the complexity of deciding the $(2,1)$-C1P is the only one that remains. Indeed, if deciding the $(2,1)$-C1P is NP-hard, it is sure that none of the constructions of the type presented in [Maňuch et al., 2011] (for showing NP-hardness of cases for larger k, δ) will work. While if it is polynomial-time decidable, I think the best place to start would be expand the forbidden substructure characterization for the C1P [Tucker, 1972] to such a characterization for the $(2,1)$-C1P.

References

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