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Acyclic 3-Colorings and 4-Colorings of Planar Graph Subdivisions

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An *acyclic coloring* of a graph G is an assignment of colors to the vertices of G such that no two adjacent vertices receive the same color and every cycle in G has vertices of at least three different colors. An *acyclic k-coloring* of G is an acyclic coloring of G with at most k colors. It is NP-complete to decide whether a planar graph G with maximum degree four admits an acyclic 3-coloring [1].

Let (u, v) be an edge of G. *Subdividing* edge (u, v) is the operation of replacing the edge with a path $u, w_1, \ldots, w_x, v$, where each w_i, $1 \leq i \leq x$, is a vertex of degree two. We call each vertex w_i, $1 \leq i \leq x$, a *division vertex*. A graph G' is a *subdivision* of another graph G, if G' is obtained by subdividing some edges of G.

Wood proved that every graph has a subdivision with two division vertices per edge that is acyclically 3-colorable [5]. Angelini and Frati showed that every triangulated planar graph with n vertices has a subdivision with one division vertex per edge that is acyclically 3-colorable and thus the total number of division vertices is $3n-6$ [1]. Mondal et al. proved that every triangulated planar graph with n vertices has a subdivision with at most one division vertex per edge that is acyclically 3-colorable (4-colorable), where the total number of division vertices is $2n - 5$ ($1.5n - 4.5$) [4].

Acyclic colorings of planar graphs have been used to obtain upper bounds on the volume of 3-dimensional straight-line grid drawings of planar graphs [2]. Consequently, acyclic colorings of planar graph subdivisions can give upper bounds on the volume of 3-dimensional polyline grid drawings, where the number of division vertices corresponds to the number of bends in the drawing. As another example, solving large scale optimization problems often makes use of sparse forms of Hessian matrices; acyclic coloring provides a technique to compute these sparse forms [3].

Under these circumstances, we ask the following question.

Open Problem: *What is the minimum positive constant c such that every triangulated planar graph with n vertices has an acyclic k-coloring, $k \in \{3, 4\}$, with at most cn division vertices?*

References

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