Readability of bipartite graphs

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presented at IWOCA 2015
16-10-2015

1 The problem, briefly

Given two finite sets of finite strings $S, T$, the bipartite overlap graph of $(S, T)$ is the bipartite graph $B(S, T)$ with parts $\{u_s : s \in S\}$ and $\{v_t : t \in T\}$ such that $u_s$ is adjacent to $v_t$ if and only if strings $s$ and $t$ overlap (that is, some nonempty suffix of $s$ equals some prefix of $t$). It is known that every bipartite graph is isomorphic to some bipartite overlap graph. Therefore, we can define the readability of a bipartite graph $G$ as

$$r(G) = \min \left\{ \max_{s \in S \cup T} |s| : G \cong B(S, T) \right\},$$

where $\cong$ denotes the graph isomorphism relation.

Open problem ([2]): Determine the computational complexity of the problem of computing the readability of a given bipartite graph $G$.

2 The problem, in some more detail

Let $G = (V, E)$ be a bipartite graph with a given bipartition of its vertex set $V(G) = V_s \cup V_p$. (We will also use the notation $G = (V_s, V_p, E)$. A labeling of $G$ is a function $\ell$ assigning a string to each vertex such that all strings have the same length, denoted by $\text{len}(\ell)$. Given two strings $x$ and $y$, we say that $x$ overlaps $y$ if there is a nonempty suffix of $x$ that is equal to a nonempty prefix of $y$. An overlap labeling of $G$ is a labeling $\ell$ of $G$ such that for all $u \in V_s$ and $v \in V_p$, $(u, v) \in E$ if and only if the strings $\ell(u)$ and $\ell(v)$ overlap (that is, if some nonempty suffix of $\ell(u)$ equals some nonempty prefix of $\ell(v)$).

The readability of $G$, denoted by $r(G)$, is the smallest nonnegative integer $r$ such that there exists an overlap labeling of $G$ of length $r$. It follows from results of Braga and Meidanis [1] that
every bipartite graph $G$ has $r(G) \leq 2^{\Delta(G)} - 1$, where $\Delta(G)$ denotes the maximum degree of a vertex in $G$ (see [2]). (In fact, every bipartite graph admits an overlap labeling over the binary alphabet.)

**Problem 1** (Chikhi et al. [2]) *Determine the computational complexity status of computing the readability of a given bipartite graph $G = (V_s, V_p, E)$.*

Remarks: The cases $r(G) \leq 1$ and $r(G) \leq 2$ are known to be polynomial. For every constant $k \geq 3$, the complexity of the problem of testing if $r(G) \leq k$ is open. Note that there is no restriction on alphabet size. (However, problems analogous to Problem 1 seem to be open also for fixed-size alphabets.)

**References**
