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Hamiltonian Paths in the Complete Graph

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Let v be an odd integer and let K_v be the complete graph with vertex-set $V = \{0, 1, \ldots, v-1\}$. We define the length of an edge $[x, y]$ of K_v as

$$l(x, y) = \min\{|x - y|, v - |x - y|\}.$$

Given an arbitrary subgraph G of K_v, the list of edge-lengths of G is

$$l(G) = \{l(x, y) : [x, y] \in E(G)\}.$$

If L is a multiset consisting of $v - 1$ elements, then we would like to find an Hamiltonian path using edges whose lengths are exactly the elements of L.

It is not hard to prove that such a path does not exist if v is not a prime and L is a well chosen multiset (basically if L contains too many elements which are not coprime with v). Buratti conjectured in 2007 that this is never the case if v is a prime.

Conjecture 1 (Buratti (2007)) *Given a prime $p = 2n+1$ and a multiset L of $2n$ elements taken from $\{1, 2, \ldots, n\}$, there exists an Hamiltonian path H in K_p such that $l(H) = L$.*

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