

# Does P-complete have a hierarchy like NP-complete has $W[*]$ ?

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The area of *Parameterized Complexity* has witnessed tremendous growth in the last two decades and has become a central research area in theoretical computer science. A problem is *fixed-parameter tractable* (FPT) if it has an algorithm that runs in  $\mathcal{O}(f(k) \cdot n^{\mathcal{O}(1)})$ , where  $n$  is the problem size and  $k$  is the input parameter that is independent of  $n$ , for an arbitrary computable function  $f$ . Numerous other NP-complete problems fall into the category of FPT but not all. It has been shown that there is an  $W[*]$  hierarchy in NP-complete, and  $\text{FPT} = W[0]$ .

The study of Parameterized Complexity has been extended to parallel computing and this is broadly known as *Parameterized Parallel Complexity*. The first systematic study of Parameterized Parallel Complexity appeared in [2]. A number of problems, unfortunately also not all, do have an NC algorithm after fixed one (or more) nontrivial parameter. For instance, the graph genus problem [3], and the monotone circuit value problem [1]. Actually we already gave a refine class called *fixed-parameter parallel tractable* (FPPT) to characterize the problems that can be efficiently parallelized, which contains all parameterized problems that have a parallel deterministic algorithm with running time  $\mathcal{O}(f(k) \cdot (\log n)^\alpha)$  and using  $\mathcal{O}(n^\beta)$  parallel processors, where  $n$  is the problem size,  $k$  is the parameter,  $f$  is an arbitrary computable function, and  $\alpha, \beta$  are constants independent of  $n$  and  $k$ .

Then we ask the following question, if restrict our attention to P-complete, does it also have a hierarchy (we call  $Z[*]$  temporarily) like  $W[*]$  in NP-complete, such that  $\text{FPPT} = Z[0]$ ?

## References

- [1] F. N. Abu-Khzam, S. Li, C. Markarian, F. Meyer auf der Heide, and P. Podlipyan. The monotone circuit value problem with bounded genus is in nc. In *International Computing and Combinatorics Conference*, pages 92–102. Springer, 2016.

- [2] M. Cesati and M. Di Ianni. Parameterized parallel complexity. In *Euro-Par98 Parallel Processing*, pages 892–896. Springer, 1998.
- [3] M. Elberfeld and K.-i. Kawarabayashi. Embedding and canonizing graphs of bounded genus in logspace. In *Proceedings of the 46th Annual ACM Symposium on Theory of Computing*, pages 383–392. ACM, 2014.