International Workshop On Combinatorial Algorithms
IWOCA

http://www.iwoca.org—Problems section.

Graphs with no equal length cycles

Chunhui Lai
School of Mathematics and Statistics,
Minnan Normal University, Zhangzhou,
Fujian 363000, CHINA
laichunhui@mnnu.edu.cn; laich2011@msn.cn

presented at IWOCA 2007
Updated Oct. 2015

Let $f(n)$ be the maximum number of edges in a graph on n vertices in which no two cycles have the same length. In 1975, Erdös raised the problem of determining $f(n)$ (see J.A. Bondy and U.S.R. Murty [1], p.247, Problem 11). Y. Shi [10] proved that

$$f(n) \geq n + [(\sqrt{8n-23}+1)/2]$$

for $n \geq 3$. E. Boros, Y. Caro, Z. Füredi and R. Yuster [2] proved that

$$f(n) \leq n + 1.98\sqrt{n}(1+o(1)).$$

Chunhui Lai [7] proved that

$$\liminf_{n\to\infty} \frac{f(n)-n}{\sqrt{n}} \geq \sqrt{2.4}.$$

and conjectured that

$$\lim_{n\to\infty} \frac{f(n)-n}{\sqrt{n}} = \sqrt{2.4}.$$

We think it is difficult to prove this conjecture. It would be nice to prove the following conjecture:

Conjecture [6].

$$\liminf_{n\to\infty} \frac{f(n)-n}{\sqrt{n}} \leq \sqrt{3}.$$

Let $f_2(n)$ be the maximum number of edges in a 2-connected graph on n vertices in which no two cycles have the same length.

In 1988, Shi[10] proved that

For every integer $n \geq 3$, $f_2(n) \leq n + [\frac{1}{2}(\sqrt{8n-15}-3)]$.

In 1998, Guantao Chen, Jeno Lehel, Michael S.Jacobson, and Warren E. Shreve [3] proved that $f_2(n) \geq n + \sqrt{n/2} - o(\sqrt{n})$

In 2001, E. Boros, Y. Caro, Z. Füredi and R. Yuster [2] improved this lower bound significantly: $f_2(n) \geq n + \sqrt{n} - O(n^{\frac{9}{20}})$.

and conjectured that

$\lim \frac{f_2(n)-n}{\sqrt{n}} = 1$.

It is easy to see that this Conjecture implies the (difficult) upper bound in the Erdös-Turan Theorem [4][5](see [2]).

Markström [9] raised the problem of determining the maximum number of edges in a hamiltonian graph on n vertices with no repeated cycle lengths.

The survey article on this problem can be found in [8].

References

[1] J.A. Bondy and U.S.R. Murty. Graph Theory with Applications (Macmillan, New York, 1976).

[2] E. Boros, Y. Caro, Z. Füredi and R. Yuster, Covering non-uniform hypergraphs, Journal of Combinatorial Theory, Series B 82(2001), 270-284.

[3] Guantao Chen, Jeno Lehel, Michael S.Jacobson, and Warren E. Shreve, Note on graphs without repeated cycle lengths, Journal of Graph Theory, 29(1998),11-15.

[4] P. Erdös , On a problem of Sidon in additive number theory and on some related problems. Addendum, J. Lond. Math. Soc. 19 (1944), 208.

[5] P. Erdös and P. Turan, On a problem of Sidon in additive number theory, and on some related problems, J. Lond. Math. Soc. 16 (1941), 212–215.

[6] Chunhui Lai, On the size of graphs with all cycle having distinct length, Discrete Math. 122(1993) 363-364.

[7] Chunhui Lai, Graphs without repeated cycle lengths, Australasian Journal of Combinatorics 27(2003), 101-105.

[8] Chunhui Lai, Mingjing Liu, Some open problems on cycles, Journal of Combinatorial Mathematics and Combinatorial Computing,91 (2014), 51-64. arXiv:1110.1144v3.

[9] K. Markström, A note on uniquely pancyclic graphs, Australasian Journal of Combinatorics 44 (2009), 105-110.

[10] Yongbing Shi, On maximum cycle-distributed graphs, Discrete Math. 71(1988) 57-71.