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Complexity of Magic Labeling

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Let an undirected graph $G = (V, E)$ be given with vertex set V and edge set E, where $|V| = n$ and $|E| = m$. A *labeling* is a one-to-one mapping $\lambda : V \cup E \to \{1, 2, \ldots, m + n\}$. For a given labeling define the *weight* $\omega(e)$ of an edge $e \in E$ as the sum of the label of e and of the labels of its two endpoints. An *edge-magic labeling (EML)* is a labeling λ for which a constant $k \in \mathbb{N}$ exists such that $\omega(e) = k$ for each edge $e \in E$. Define the *weight* $\omega(v)$ of a vertex $v \in V$ as the sum of the label of v and of the labels of all edges incident to v. A *vertex-magic labeling (VML)* is a labeling λ for which a constant $h \in \mathbb{N}$ exists such that $\omega(v) = h$ for each vertex $v \in V$. A *totally magic labeling (TML)* is a labeling λ for which (not necessarily equal) constants $h, k \in \mathbb{N}$ exist such that λ is edge-magic with constant k and vertex-magic with constant h.

Although magic labellings have been extensively studied, there are very few papers dealing with their algorithmic or complexity issues (see [1], for instance). To our knowledge, the complexity of the three problems to decide, whether there exists an EML with given constant $k \in \mathbb{N}$, a VML with given constant $h \in \mathbb{N}$, and a TML with given constants $h, k \in \mathbb{N}$, is unknown.

References

[1] Gerold Jäger. SAT and IP Based Algorithms for Magic Labeling with Applications. *Proceedings of IWOCA 2013*.