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Wheeler Graph Recognition on 3-NFAs and 4-NFAs

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Wheeler graphs were defined by Gagie *et al.,* in [2] to capture important properties of several objects which can be indexed using Burrows-Wheeler Transform (BWT) based data structures. These objects include labeled trees, de Bruijn graphs, and the multi-string BWT, to name a few. A Wheeler graph is a directed graph where each edge is labeled by a character from a totally-ordered alphabet, and there exists an ordering of its vertices that satisfies the following properties: The vertices with in-degree 0 precede those with positive in-degree, and for any pair of edges $e = (u, v)$ and $e' = (u', v')$ labeled a and a' respectively,

- $a \prec a' \implies v < v'$,
- $(a = a') \wedge (u < u') \implies v \leq v'$.

The problem of determining whether a general graph is a Wheeler graph was proven NP-complete in [3], even for directed acyclic graphs with edge labels over a binary alphabet. However, the computational complexity of this problem remains open for many interesting classes of graphs.

We call a directed graph where each edge is labeled with a character and at most d edges with any particular label leave from any vertex a d-NFA. The problem of recognizing if a 2-NFA is a Wheeler graph is shown to be solvable in polynomial time in [1]. On the other hand, the problem of recognizing if a 5-NFA is a Wheeler graph is shown to be NP-complete in [3]. This leaves the complexity of the Wheeler graph recognition problem for 3-NFAs and 4-NFAs as an intriguing open question. What is the computational complexity of recognizing whether a 3-NFA or 4-NFA is a Wheeler graph?

References

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