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The complexity of covering a ladder using cycles

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The *ladder graph of length* n, L_n, is the cartesian product $P_2 \square P_n$. We note that L_n contains $\binom{n}{2}$ distinct cycles. We define the following decision problem:

LADDER CYCLE COVER
INSTANCE: An integer n, an integer $m \leq \binom{n}{2}$, an integer $k \leq m$, and a set $\mathcal{C}$ of m cycles of the *ladder graph* L_n.
QUESTION: Is there a set $\mathcal{C}' \subseteq \mathcal{C}$ of k cycles such that the union of all cycles of $\mathcal{C}'$ covers L_n, i.e. $\bigcup_{C_i \in \mathcal{C}'} E(C_i) = E(L_n)$?

Open problem 1 *What is the complexity of the decision problem* LADDER CYCLE COVER*? What if we restrict it to instances having* $|\mathcal{C}| = n$*?*

This problem, together with further discussions, is also mentioned in my PhD dissertation [2, Section 8.1]. Let us make a few observations.

Observation 1 *Of course, the answer to the question can be "YES" only if* $\mathcal{C}$ *itself is a valid solution, i.e.* $\bigcup_{C_i \in \mathcal{C}} E(C_i) = E(L_n)$*. This can be checked easily beforehand.*

Observation 2 *Since each cycle covers exactly two "step edges" of* L_n *(the ones coming from the* P_2 *fiber), we need at least* $\frac{n}{2}$ *cycles in the solution.*

Observation 3 *(1) If we were asking only to cover the edges of* L_n *coming from the* P_n *fiber, the problem would correspond to* DOMINATING SET *in some interval graph, which is solvable in linear time [1].*
(2) If we were asking only to cover the edges of L_n *coming from the* P_2 *fiber ("step edges"), the problem would correspond to* EDGE COVER *in a graph, which is solvable in polynomial time by a maximum matching approach [4] (cited in [3, Problem GT1]).*

Note that we can equivalently state LADDER CYCLE COVER as follows:

INTERVAL AND ENDPOINTS COVER
INSTANCE: An integer n, an integer $m \leq \binom{n}{2}$, an integer $k \leq m$, and a set $\mathcal{C} = \{\{s_1, t_1\}, \ldots, \{s_m, t_m\}\}$ of m pairs from the set $\{1, \ldots, n\}$.
QUESTION: Is there a set $\mathcal{C}' \subseteq \mathcal{C}$ of size k such that for each element i of $\{1, \ldots, n\}$:
(1) if $i < m$, the interval $[i, i+1]$ is included in some interval $[s_i, t_i]$ defined by a pair in $\mathcal{C}'$, and
(2) i belongs to some pair of $\mathcal{C}'$?

References

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[2] F. Foucaud. *Combinatorial and algorithmic aspects of identifying codes in graphs*. PhD thesis, Université Bordeaux 1, France, December 2012. Available online at `http://tel.archives-ouvertes.fr/tel-00766138`.

[3] M. R. Garey and D. S. Johnson. *Computers and intractability: a guide to the theory of NP-completeness*, W. H. Freeman, 1979.

[4] E. L. Lawler. *Combinatorial optimization: networks and matroids*, Holt, Rinehart and Winston, 1976.