International Workshop On Combinatorial Algorithms
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Vector connectivity with demands at most 3

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presented at IWOCA 2015
16-10-2015

Given a (finite, simple, undirected) graph $G = (V, E)$, a set $S \subseteq V$ and a vertex $v \in V \setminus S$, a v–S *fan of order* k is a collection of k paths $P_1, \ldots, P_k$ such that (1) every P_i is a path connecting v to a vertex of S, and (2) the paths are pairwise vertex-disjoint except at v, i.e., for all $1 \leq i < j \leq k$, it holds that $V(P_i) \cap V(P_j) = \{v\}$. Given a graph $G = (V, E)$ and an integer-valued function $r : V \to \mathbb{Z}_+ = \{0, 1, 2, \ldots\}$, a *vector connectivity set* for (G, r) is a set $S \subseteq V$ such that for every $v \in V \backslash S$, there exists a v–S fan of order $r(v)$. We refer to $r(v)$ as the *demand* (or *requirement*) of vertex v.

In [1], Boros et al. introduced the VectorConnectivity (VecCon) problem as the problem of finding the minimum size of a vector connectivity set for (G, r). (Note that if we require each path to be of length exactly 1, we get the well-known Vector Domination problem (see, e.g., [3, 4]), which is a generalization of the Dominating Set and Vertex Cover problems.)

Let $R(G, r) = \max_{v \in V(G)} r(v)$. Cicalese et al. [2] showed that the VecCon is APX-hard on instances with $R(G, r) \leq 4$ and polynomially solvable if $R(G, r) \leq 2$.

Problem 1 (Cicalese et al. [2]) *What is the complexity of* VecCon *on instances* (G, r) *such that* $R(G, r) = 3$*?*

For more (algorithmic) open problems related to vector connectivity refer to [2].

References

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