A context-free grammar $G = (N, \Sigma, R, S)$ is a singleton grammar if $R$ is a total function $N \rightarrow (N \cup \Sigma)^+$ and the relation $\{(A, B) \mid (A, w) \in R, |\alpha|_B \geq 1\}$ is acyclic. The language corresponding to such a grammar $G$ contains only a single word, denoted by $D(G)$. The size of $G = (N, \Sigma, R, S)$ is given by $|G| = \sum_{(A, w) \in R} |w|$.

**Shortest Grammar Problem (SGP)**

*Instance:* A word $w \in \Sigma^+$ and a $k \in \mathbb{N}$.

*Question:* Does there exist a grammar $G$ with $D(G) = w$ and $|G| \leq k$?

Known results, open for improvement:

- NP-hardness for alphabet-size $|\Sigma| = 24$, possibly improvable to $|\Sigma| = 18$ (see [1]) but especially binary alphabet open.

- $\log(|w|)$-approximation for any, especially unbounded, terminal alphabet (see [2, 3]). Constant-factor approximation (for bounded alphabet)?

- W[1]-hardness for parameter $|N|$ (number of non-terminals) for unbounded terminal alphabet $\Sigma$. Fixed-parameter tractability with parameter $|N|$ for bounded $\Sigma$ only known to be in XP (see [1]) but otherwise open.

**References**
