Distance Pattern Distinguishing Sets in Graphs

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Definition 1. *Let $G = (V, E)$ be a connected (p, q)-graph. Let M be a nonempty subset of $V(G)$ and let $u \in V(G)$. The M-distance pattern of u is the set $f_M(u) = \{d(u, v) : v \in M\}$. If f_M is an injective function, then the set M is called a distance pattern distinguishing set (or DPD-set in short) of G.*

Observation 2. *For any graph $G = (V, E)$ with $|V| \geq 2, V$ is not a DPD-set, since for any two vertices $u, v \in V$ with $d(u, v) = diam(G)$ we have $f_M(u) = f_M(v) = \{0, 1, 2, \ldots, diam(G)\}$.*

We observe that not every graph has a DPD-set. For example the complete graph $K_n, n \geq 3$ does not possess a DPD-set. Also any tree with a support vertex having at least three pendant vertices as its neighbours does not possess a DPD-set.

Problem

1. Characterize graphs which admit a DPD-set.
2. Given a positive integer $k \geq 3$, does there exist a k-regular graph which admits a DPD-set?