Deterministic k-Local Tree Automata and their Work-optimal Parallel Run

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Overview

Synchronizing word

- Property of deterministic finite string automata
- Reading such word from any configuration puts the automaton in a well defined state

input		a	b	**a**	**b**	**c**	c	a	**a**	**b**	**c**	a	c	b	**a**	**b**	**c**	b	a	c	**a**	**b**	**c**
state	0	1	2	1	2	**3**	0	1	1	2	**3**	1	0	0	1	2	**3**	0	1	0	1	2	**3**

k-locality

- Stronger property
- All words of length at least k are synchronizing
- Property of pattern matching automata
  - k is the length of the pattern

k-local DFA run parallelization

Option 1

- Divide input to processors
- Start run k letters in advance of each block from an arbitrary state

k-local DFA run parallelization

Option 2

- Divide input into blocks of length k
- On each block run twice starting from an arbitrary state

Finite tree automata (bottom-up)

- Formalism for accepting regular tree languages
  - Rooted, ordered and labeled trees
  - Degree of a node is determined by its symbol
- Run is a bottom-up computation on trees with constant bounded degree

DFTA run

$A = (Q, \mathcal{F}, Q_f, \Delta)$

$Q = \{0, 1, 2, 3, 4\}$ (States)

$\mathcal{F} = \{a_2, b_1, c_0\}$ (Alphabet)

$Q_f = \{4\}$ (Final states)

Δ: (Transition function)

$c \rightarrow 1$

$b\,(0) \rightarrow 2$

$b\,(1) \rightarrow 3$

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$a\,(2, 3) \rightarrow 4$

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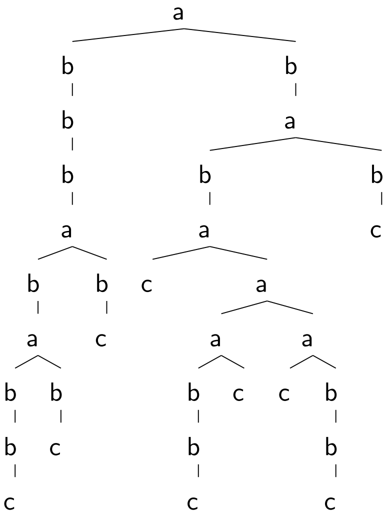
$a\,(q_1, q_2) \rightarrow 0$ otherwise



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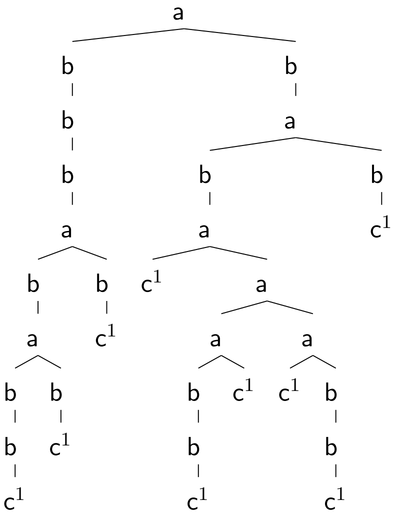

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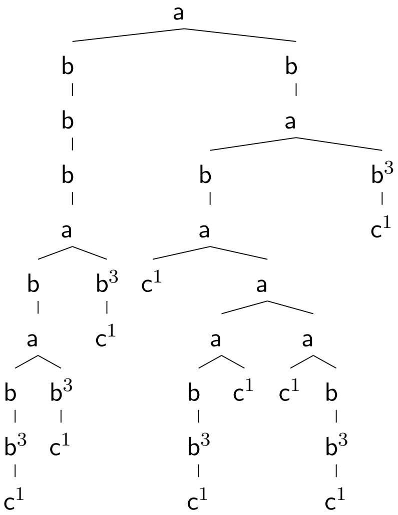

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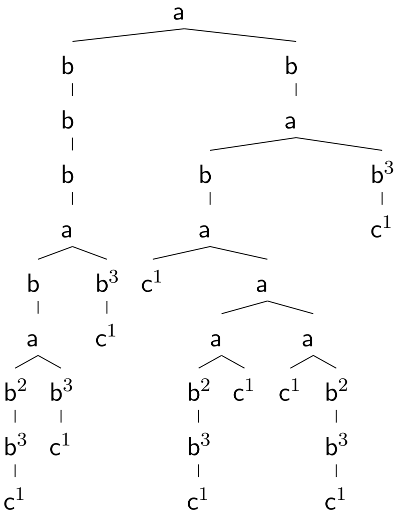

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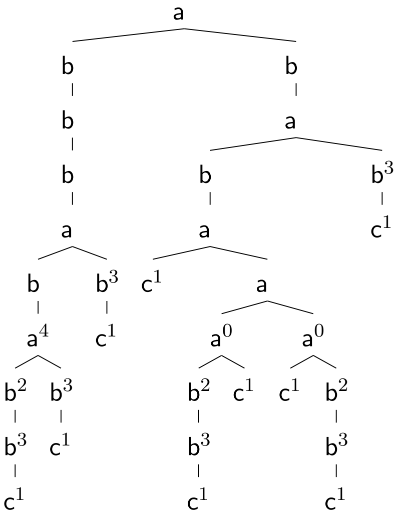

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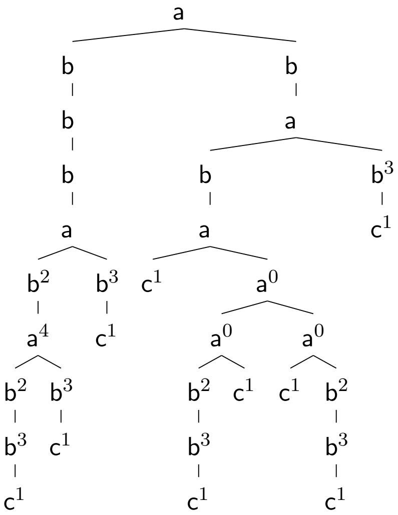

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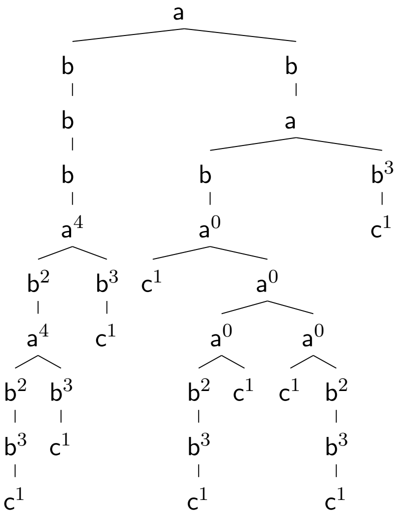

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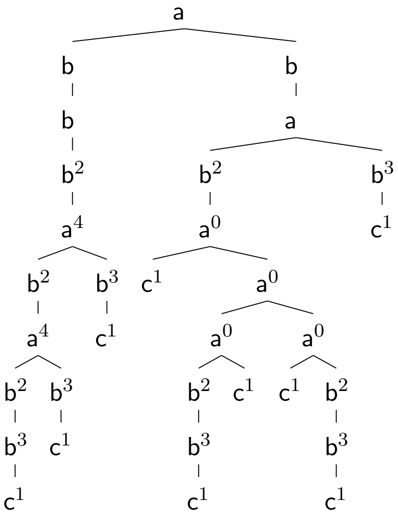

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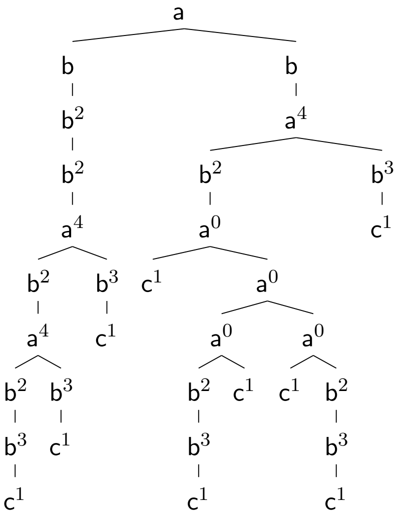

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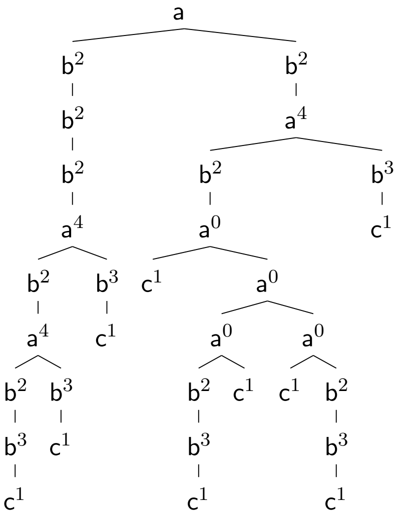

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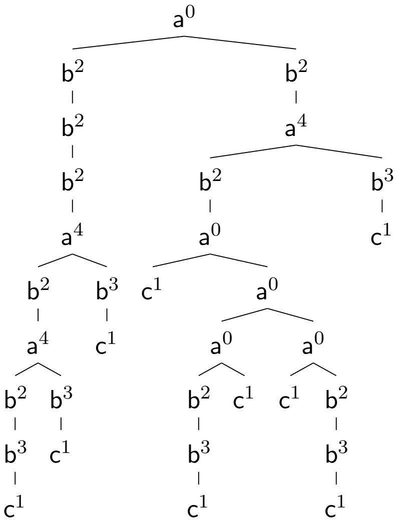


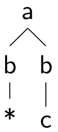
Synchronizing tree

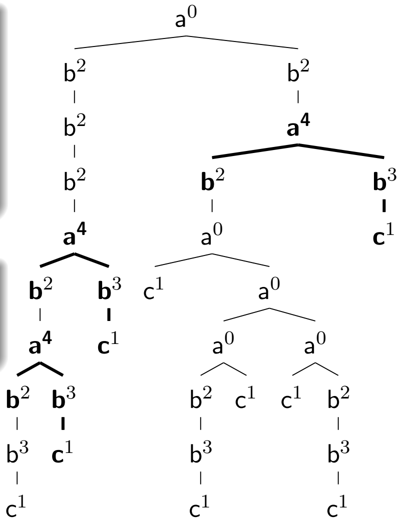
Definition

A tree pattern is *synchronizing* for a DFTA if run on the pattern with variables substituted by any tree ends in the same state.

Remark

All tree patterns without variables are synchronizing.





k-locality

Definition

A DFTA is called k-*local* if all trees with variables in depth at least k are synchronizing

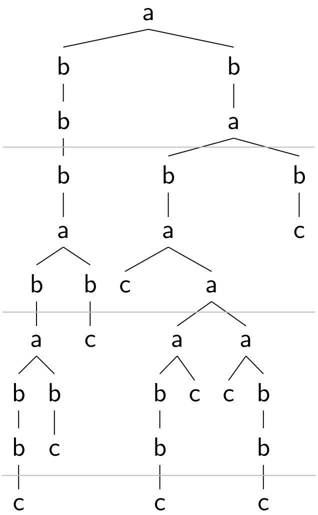
Theorem

All tree pattern matching automata are k-local, where k is the height of the pattern.

Work optimal parallel run

Basic idea

- Divide input tree into segments of k levels
- Assign each node an arbitrary state
- Perform bottom-up computation twice over each segment



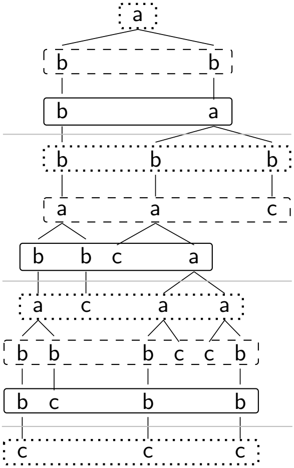
Work optimal parallel run

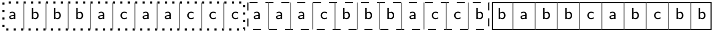
Algorithm overview

- Basic steps:
  1. Computation of a linear order of nodes
  2. Parallel traversal of ordered nodes
- Runs in $\mathcal{O}(\log n)$ time with $n/\log n$ processors on EREW PRAM
- Uses known parallel algorithms:
  - Euler tour technique
  - Parentheses matching
  - List ranking
  - Segmented prefix sum

Ordering the input tree

- All nodes with the same relative depth against their segment can be computed at the same time
- Order nodes by their relative depth
- Modified algorithm for Breath-first ordering





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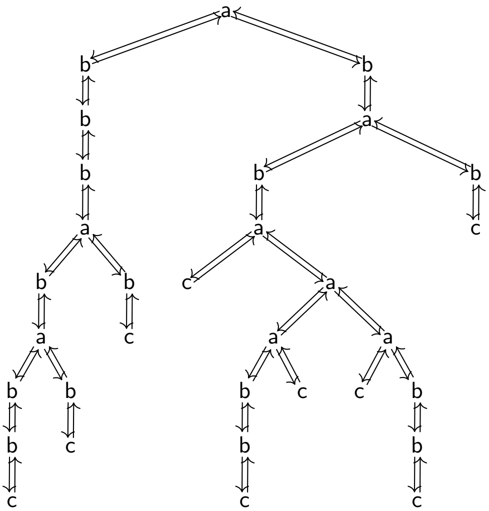

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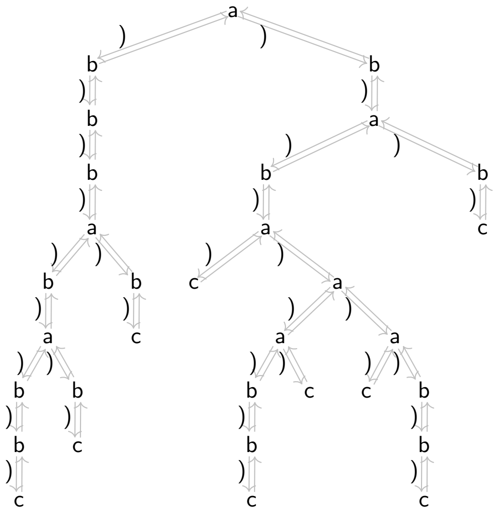

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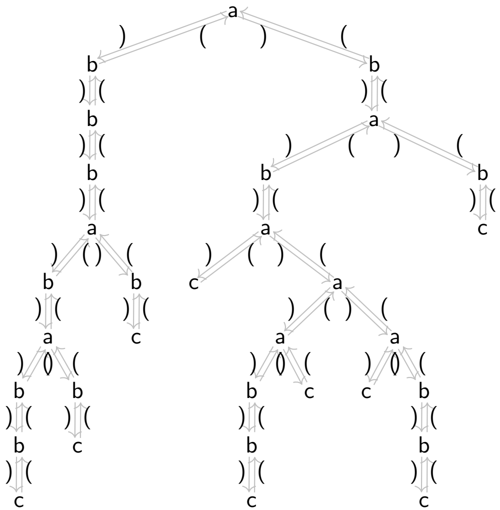

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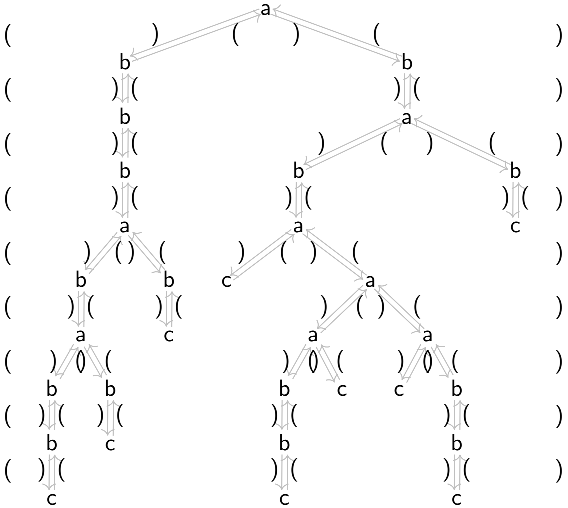

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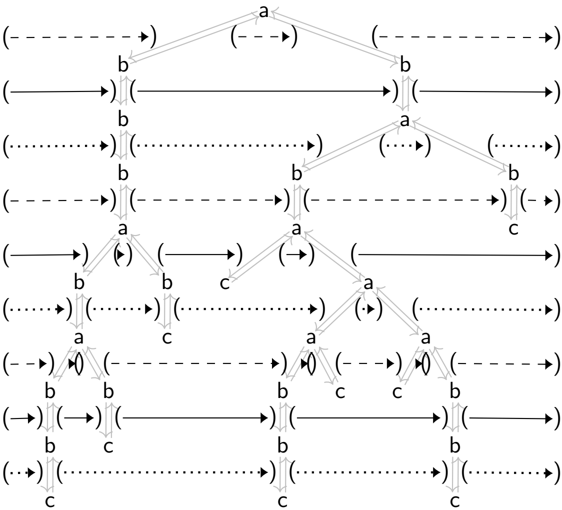

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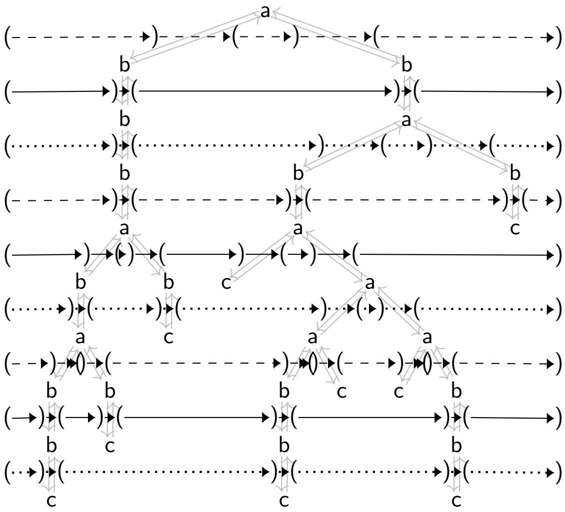

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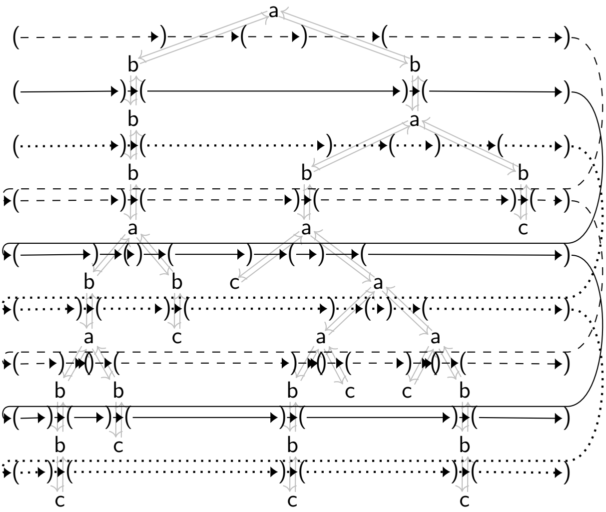

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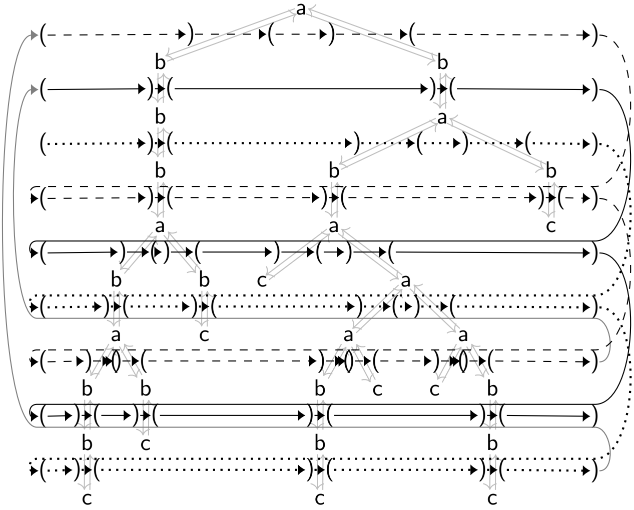

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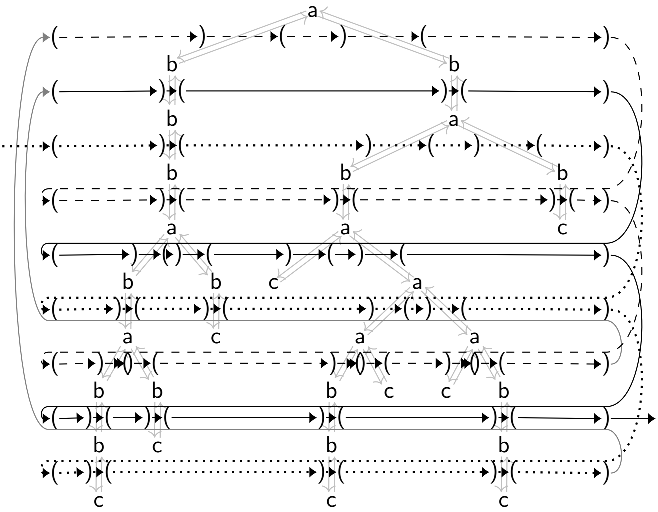

Ordered tree traversal

- Start parallel traversal from the end of the array
- All nodes from a group must be computed before moving to the next
- On the boundary some processors might stall

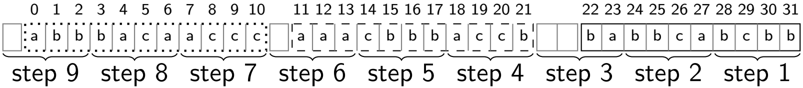

Figure: Example traversal with 4 processors

Summary

- Introduced concepts for DFTA of:
  - Synchronizing tree
  - k-locality
- Proved that tree pattern matching automata are k-local with respect to the height of the pattern
- Shown work optimal parallel algorithm for run of k-local DFTA in $\mathcal{O}(\log n)$ time with $n/\log n$ processors on EREW PRAM
  - for $k = \mathcal{O}(\log n)$
  - for general k in $\mathcal{O}(\max(k, \log n))$ time with $n/\max(k, \log n)$ processors

Thank you!