Space-efficient construction of succinct de Bruijn graphs

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Joint work with
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Outline

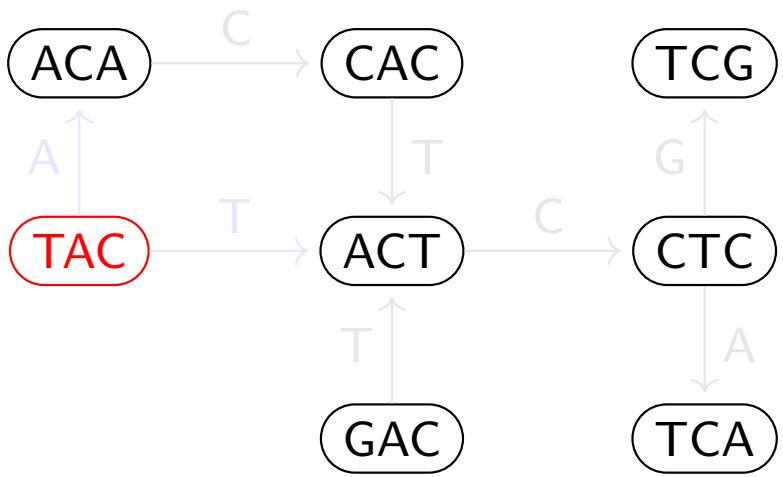
de Bruijn graphs (dBGs)

Definitions:

- Given a collection of strings $\mathcal{S}$, a *de Bruijn* graph of order k is a directed graph containing:
  - a node v for every **unique** k-**mer** $v[1]...v[k]$ in $\mathcal{S}$.
  - an edge (u, v) with label $v[k]$ if there is a $(k+1)$-**mer** $u[1]...u[k]v[k]$ in $\mathcal{S}$.

Example:

- $\mathcal{S} = \{$TACACT, TACTCA, GACTCG$\}$



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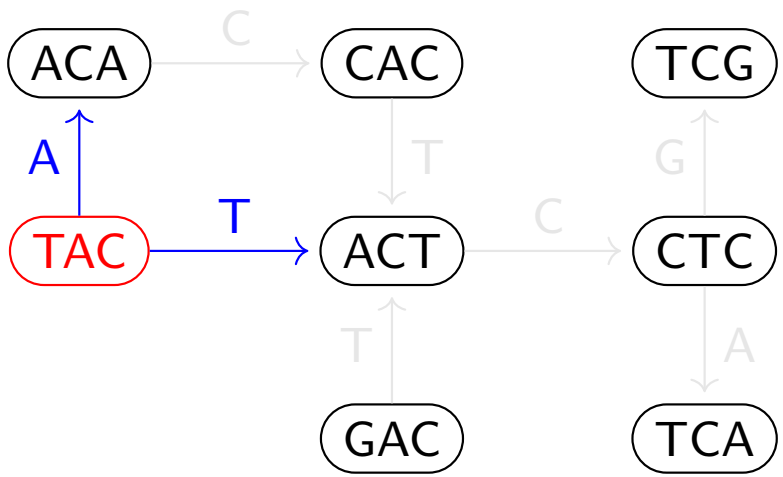

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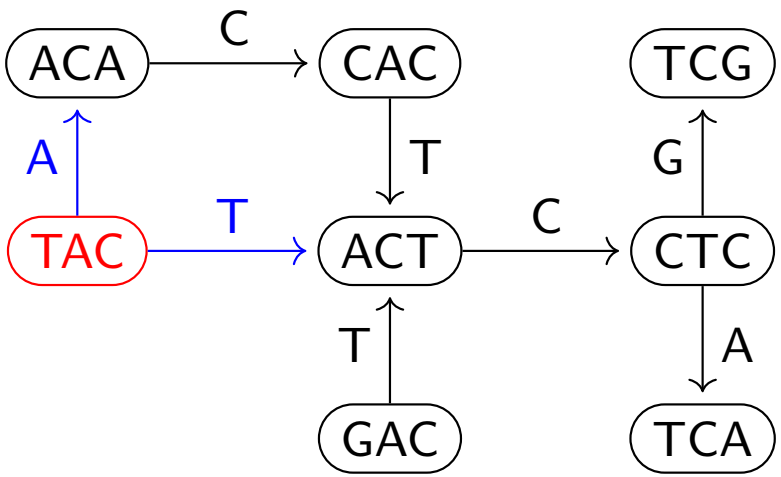

Succinct representation of dBGs:

BOSS*:

- In [Bowe *et al.*, WABI 2012] introduced a succinct representation for dBGs in space $\mathcal{O}(|E| \log \sigma)$ bits.
- BOSS representation:
  - Outgoing edges of each v_i are encoded into the substring $W_i = x_1[i]\ x_2[i]\ \ldots\ x_\sigma[i]$
  - W_i are concatenated considering the order of the reversed labels: $W = \ldots W_{\$\$\$}\, W_{\$\$T}\, W_{\$\$G}\, W_{\$TA}\, W_{ACA}\, W_{\$GA}\, W_{TAC}\, W_{CAC}\, W_{GAC}\, W_{ACT}\, W_{TCG}\, W_{CTC}\, W_{TCA} \ldots$

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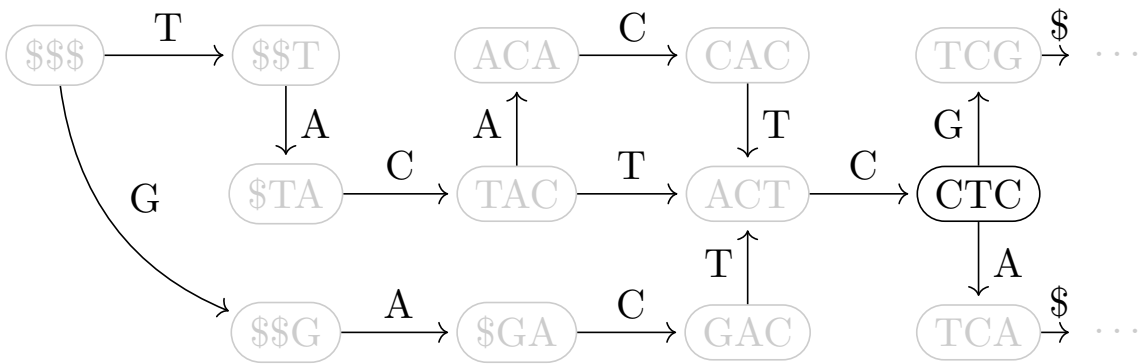

*for the authors' initials

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    $W_i = \mathsf{AT}, W_j = \mathsf{AG}$
  - W_i are concatenated considering the order of the reversed labels $\overleftarrow{v_1}, \ldots, \overleftarrow{v_n}$
    $\mathsf{TACGACT}$

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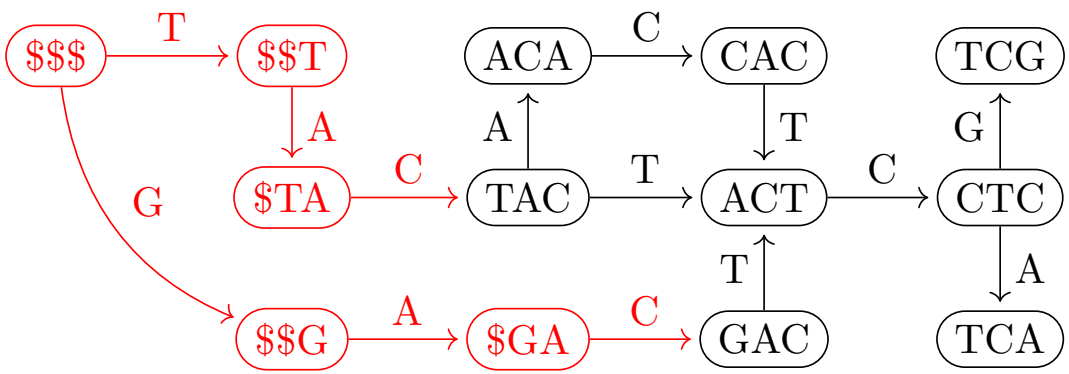
- $\mathcal{S} = \{\$\$\$\mathsf{TACACT}, \$\$\$\mathsf{TACTCA}, \$\$\$\mathsf{GACTCG}\}$



For convenience, we add k copies of a symbol \$ at the beginning of each string s_i.

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    $\overleftarrow{TAC} \prec \overleftarrow{CTC}$

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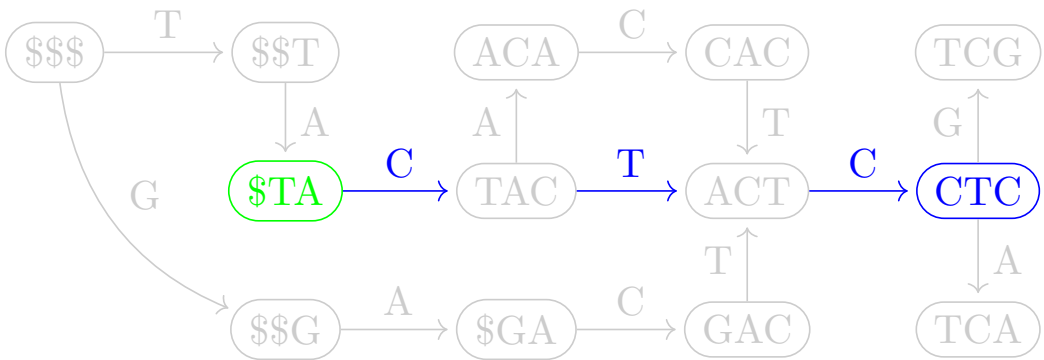

The label of every node can be recovered.

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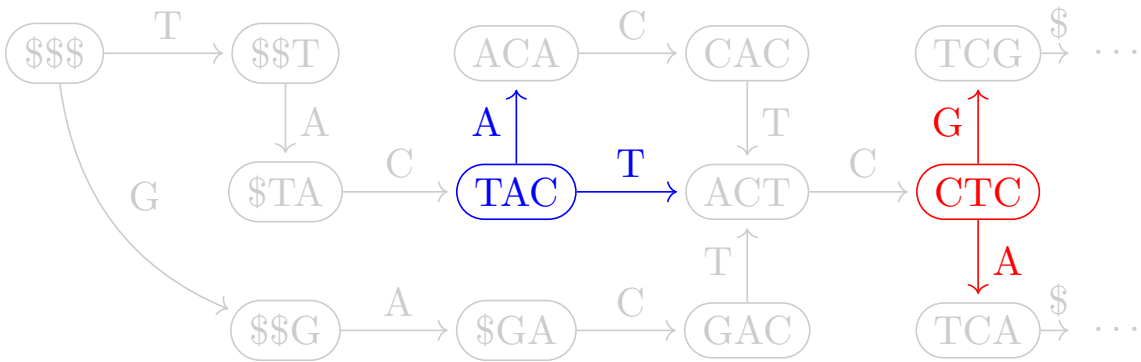

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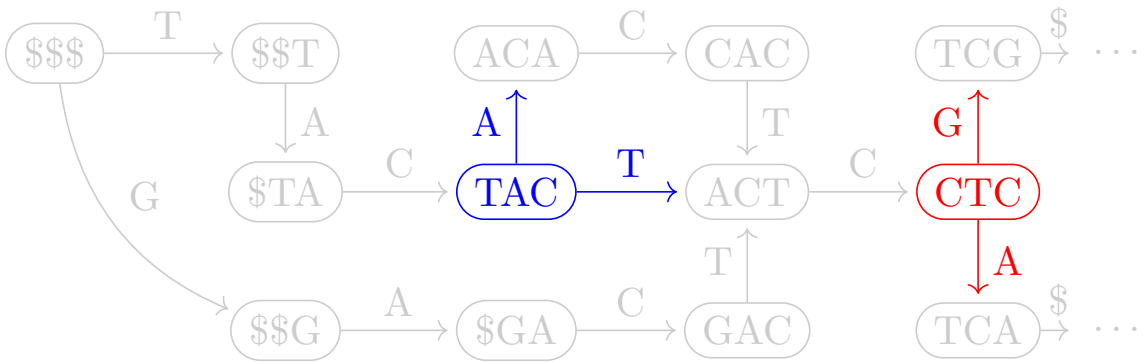

Succinct representation of dBGs:

BOSS:

- Nodes $v_i = \underline{v_i[1]...v_i[k]}$ are sorted by their **reversed labels** $\overleftarrow{v_i} = \underline{v_i[k]...v_i[1]}$
- We mark the position of the last outgoing edge of each node.
- We mark as negative ($-$) incoming edges with the same label (except the first).

last	Nodes	W
0	\$ \$ \$	G
1	\$ \$ \$	T
1	A C A	C
1	T C A	\$
1	\$ G A	C
1	\$ T A	C
1	C A C	T
1	G A C	T-
0	T A C	A
1	T A C	T-
0	C T C	A
1	C T C	G
1	\$ \$ G	A
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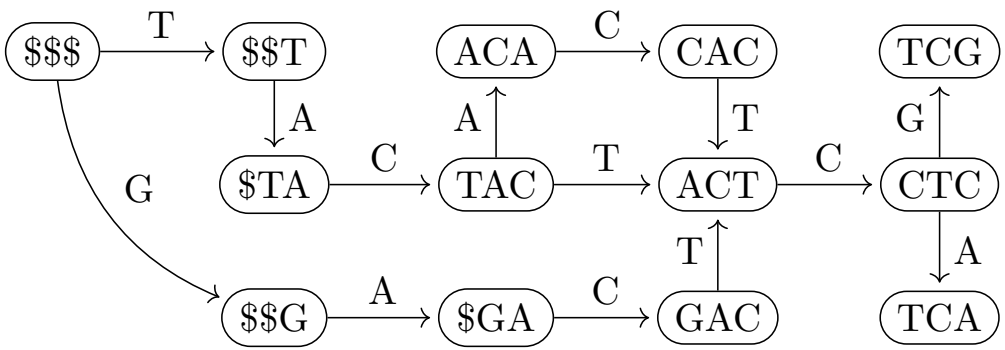

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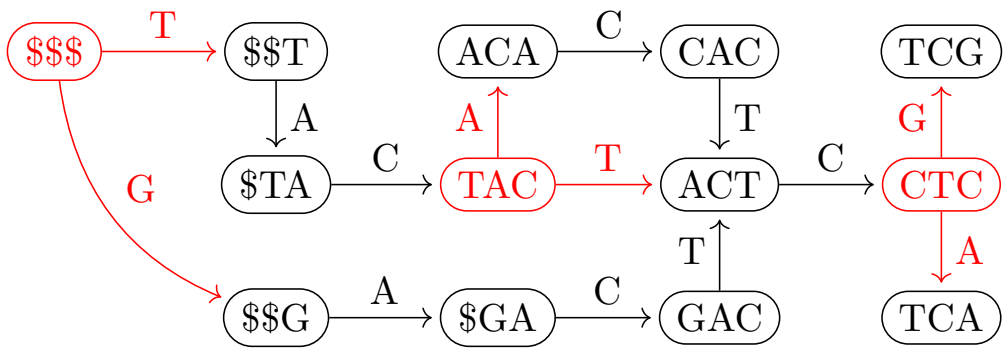

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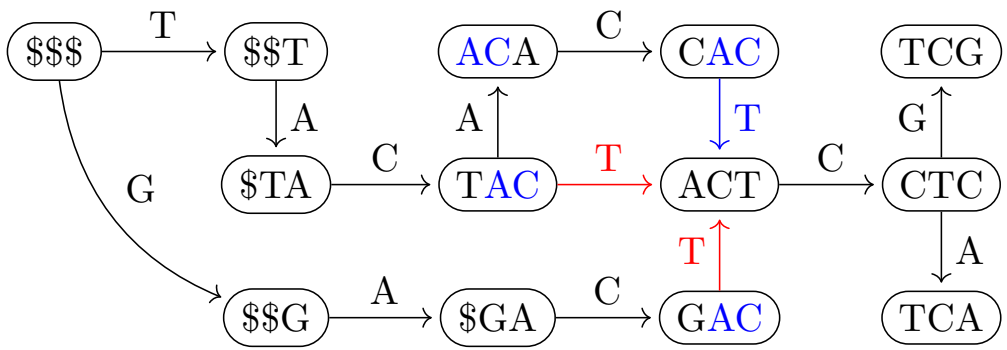
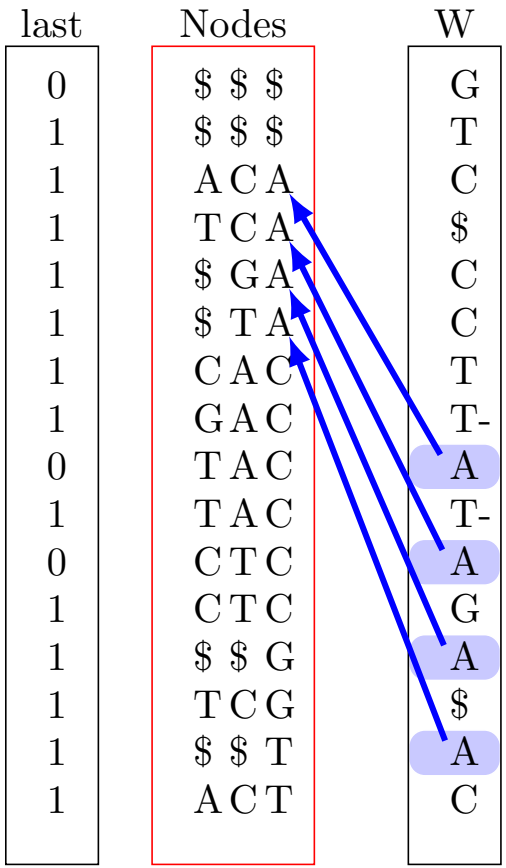
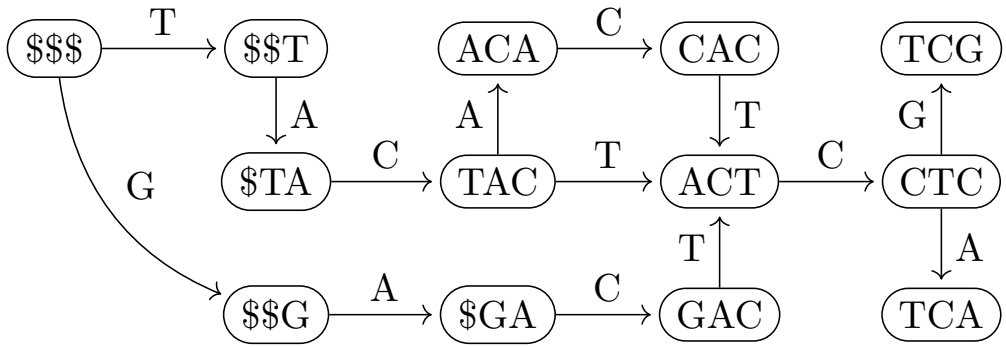


Succinct representation of dBGs:

BOSS:

- LF-mapping between the positive symbols in W and the $Nodes[k]$ (with last $= 1$).
- Fast navigation operations: Outdegree, Outgoing, Indegree and Incoming.
- Small space: $\mathcal{O}(m \log \sigma) + m + o(m)$ bits for rank and select operations.





Similar to the BWT and XBW.

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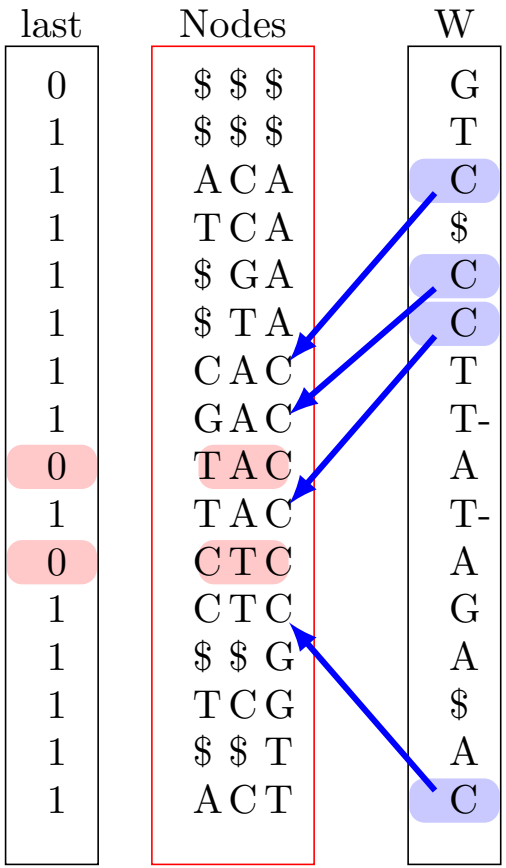

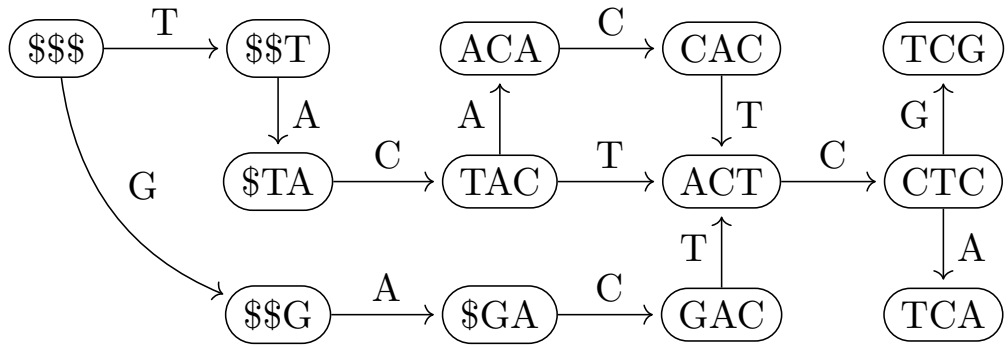

Select operation.

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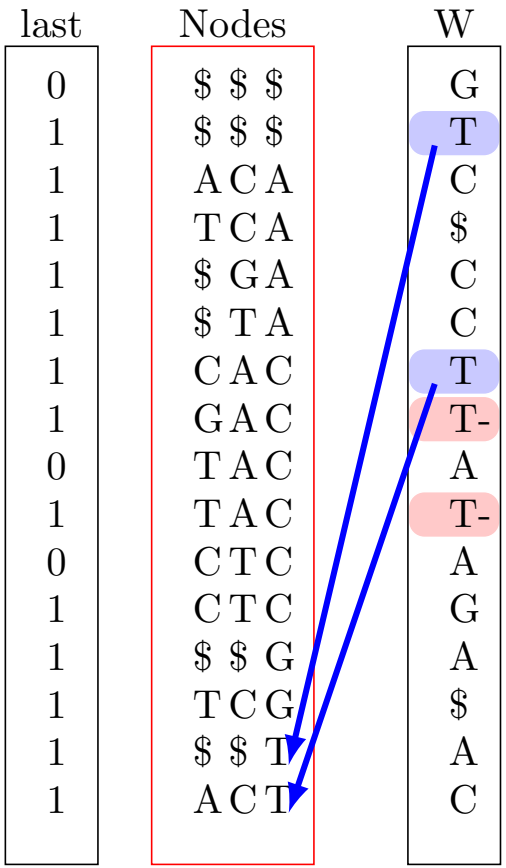

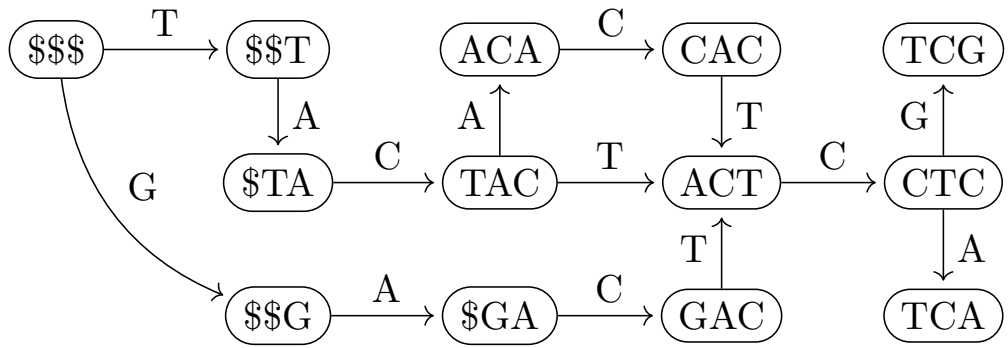

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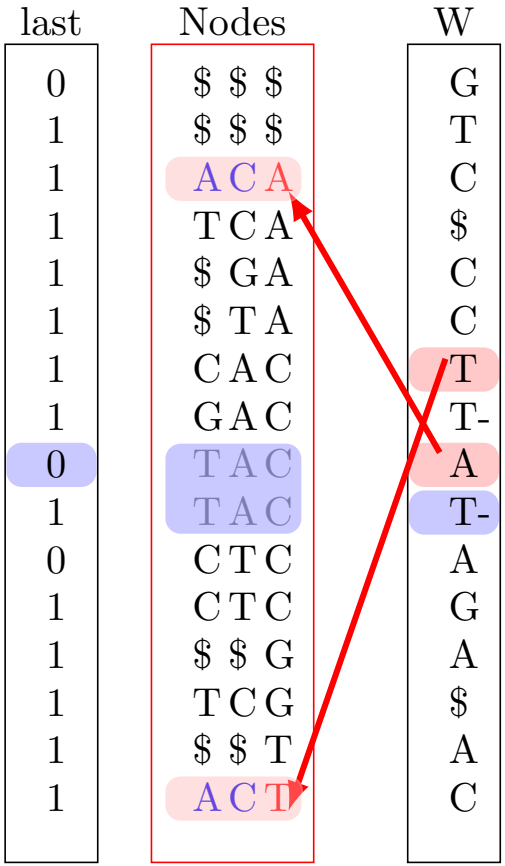

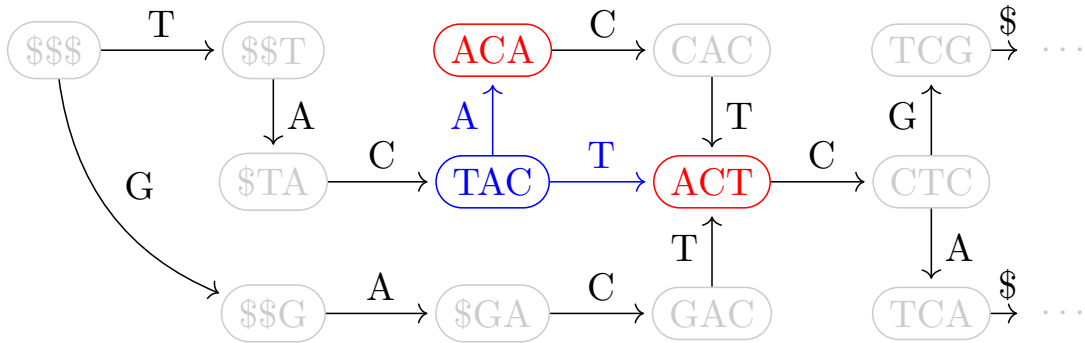

Outgoing edges.

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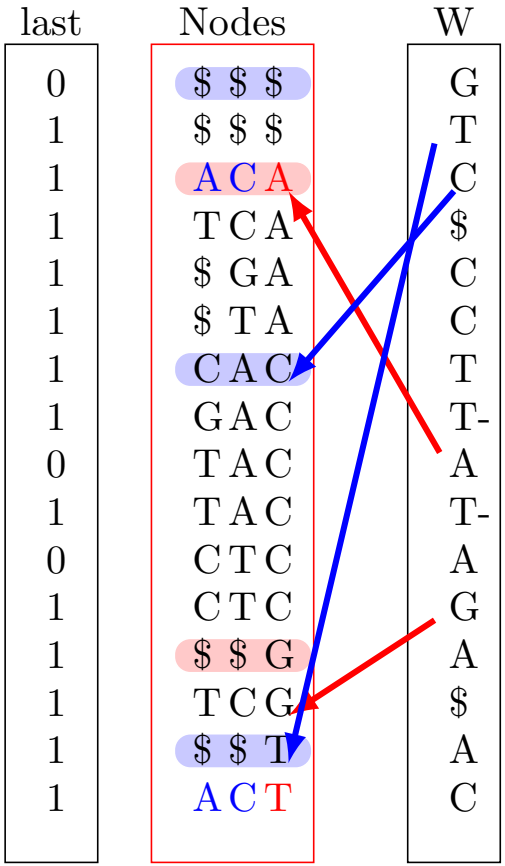

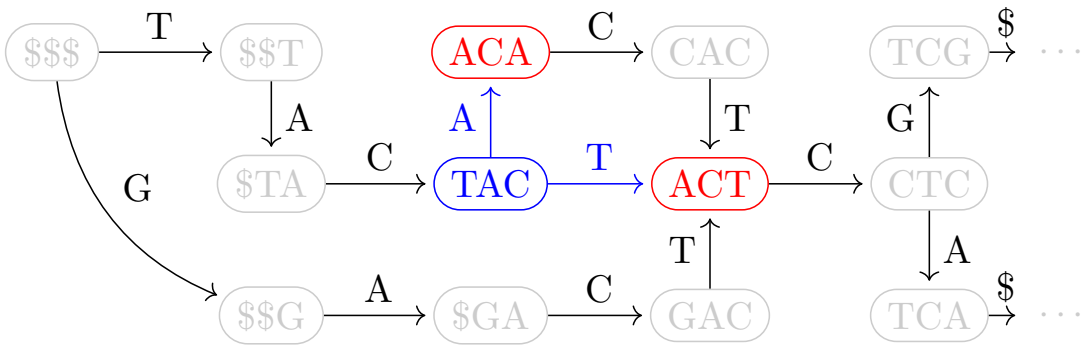

We don't need matrix Nodes, we can use counters $C[1, \sigma]$.

Outline

BOSS construction

Radix sort:

- We can sort all k-mers of collection $\mathcal{S}$, with total length N.
  - Radix sorting (from 2nd symbol of $(k+1)$-mers): in $O(N \cdot k)$ time.

$h=1$	$h=2$	$h=3$	last	Nodes	W
G $ $ $	G $ $ $	G $ $ $	0	$ $ $	G
T $ $ $	T $ $ $	T $ $ $	1	$ $ $	T
C A G $	C A C A	C A C A	1	A C A	C
C A T $	$ A C T	$ A C T	1	T C A	$
C A C A	C A G $	C A G $	1	$ G A	C
$ A C T	C A T $	C A T $	1	$ T A	C
T C A C	T C A C	T C A C	1	C A C	T
A C T C	T C A G	T C A G	1	G A C	T-
G C T C	A C A T	A C A T	0	T A C	A
T C A G	T C A T	T C A T	1	T A C	T-
A C A T	A C T C	A C T C	0	C T C	A
T C A T	G C T C	G C T C	1	C T C	G
$ G C T	$ G C T	$ G C T	1	$ $ G	A
A G $ $	A G $ $	A G $ $	1	T C G	$
A T $ $	A T $ $	A T $ $	1	$ $ T	A
C T C A	C T C A	C T C A	1	A C T	C

BOSS construction

BWT and LCP array: [Egidi et al., WABI 2018]

- Given the BWT and LCP array for the reversed strings:
  - We can compute $\langle W, \text{last} \rangle$ in $O(N)$ time: sequential scan over BWT+LCP.
  - We need only the k-truncated BWT and LCP array.

BWT	sorted suffixes	LCP
G	$ $ $	2
T	$ $ $	3
T	$ $ $	3
C	A C A T $	0
$	A C T C A T $	2
C	A G $	1
C	A T $	1
C	A T $	5
T	C A C A T $	0
T	C A G $	2
T	C A T $	2
A	C A T $	6
G	C T C A G $	1
A	C T C A T $	4
A	G $	0
$	G C T C A G $	1
A	T $ $	0
A	T $ $	4
$	T C A C A T $	1
C	T C A G $	3
C	T C A T $	3

last	Nodes	W
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1	$ T A	C
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C	A G $	1
C	A T $	1
C	A T $	5
T	C A C A T $	0
T	C A G $	2
T	C A T $	2
A	C A T $	6
G	C T C A G $	1
A	C T C A T $	4
A	G $	0
$	G C T C A G $	1
A	T $ $	0
A	T $ $	4
$	T C A C A T $	1
C	T C A G $	3
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T	C A G $	2
T	C A T $	2
A	C A T $	6
G	C T C A G $	1
A	C T C A T $	4
A	G $	0
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C	T C A G $	3
C	T C A T $	3

last	Nodes	W
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1	T C A	$
1	$ G A	C
1	$ T A	C
1	C A C	T
1	G A C	T-
0	T A C	A
1	T A C	T-
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1	$ $ G	A
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1	A C T	C

BOSS construction

Experiments:[1]

1. Counting k-mers (DSK) and Radix sorting (COSMO) $\leftarrow \mathcal{O}(N \cdot k)$ time.
2. BWT and LCP construction (egap) and BOSS construction $\leftarrow \mathcal{O}(N)$ time.

Results:

- Running time:
- Peak memory:

[1] DNA sequences of length 100 (500MB).

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Results:

- Running time: 22×, 4× and 3× slower.
- Peak memory:

RAM parameter		k=10		k=30		k=50		
		time (sec)	memory	time (sec)	memory	time (sec)	memory	
128MB	BWT+BOSS	1,486.03		1,501.62		1,496.59		← em-gap
	COSMO	**67.15**	99 MB	**397.53**	6.8 GB	**465.93**	14.4 GB	

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- Peak memory: 1.3× larger, 54× and 110× smaller.

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		time (sec)	memory	time (sec)	memory	time (sec)	memory	
128MB	BWT+BOSS	1,486.03		1,501.62		1,496.59		← em-gap
	COSMO	**67.15**	99 MB	**397.53**	6.8 GB	**465.93**	14.4 GB	

[1] DNA sequences of length 100 (500MB).

BOSS construction

Experiments:[1]

1. Counting k-mers (DSK) and Radix sorting (COSMO) $\leftarrow \mathcal{O}(N \cdot k)$ time.
2. BWT and LCP construction (egap) and BOSS construction $\leftarrow \mathcal{O}(N)$ time.

Results:

- Running time: 15×, 2.5× and 2× slower.
- Peak memory: 1.7× larger, 13.6× and 28× smaller.

RAM parameter		k=10		k=30		k=50		
		time (sec)	memory	time (sec)	memory	time (sec)	memory	
128MB	BWT+BOSS	1,486.03		1,501.62		1,496.59		← em-gap
	COSMO	**67.15**	99 MB	**397.53**	6.8 GB	**465.93**	14.4 GB	
512MB	BWT+BOSS	981.49		991.03		991.71		← se-gap
	COSMO	**65.69**	303 MB	**396.76**	6.8 GB	**479.01**	14.4 GB	

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BOSS construction

Experiments:[1]

1. Counting k-mers (DSK) and Radix sorting (COSMO) $\leftarrow \mathcal{O}(N \cdot k)$ time.
2. BWT and LCP construction (egap) and BOSS construction $\leftarrow \mathcal{O}(N)$ time.

Results:

- Running time: 7×, 1.4× and 1.2× slower.
- Peak memory: 3.5× larger, 3.4× and 7× smaller.

RAM parameter		k=10		k=30		k=50		
		time (sec)	memory	time (sec)	memory	time (sec)	memory	
128MB	BWT+BOSS	1,486.03		1,501.62		1,496.59		← em-gap
	COSMO	**67.15**	99 MB	**397.53**	6.8 GB	**465.93**	14.4 GB	
512MB	BWT+BOSS	981.49		991.03		991.71		← se-gap
	COSMO	**65.69**	303 MB	**396.76**	6.8 GB	**479.01**	14.4 GB	
2GB	BWT+BOSS	560.95		569.61		564.69		← gap
	COSMO	**78.65**	578 MB	**403.96**	6.8 GB	**479.39**	14.4 GB	

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BOSS construction

Experiments:[1]

1. Counting k-mers (DSK) and Radix sorting (COSMO) $\leftarrow \mathcal{O}(N \cdot k)$ time.
2. BWT and LCP construction (egap) and BOSS construction $\leftarrow \mathcal{O}(N)$ time.

Results:

- Running time: 22×,
- Peak memory: 1.7× larger,

RAM parameter		k=10		k=30		k=50		
		time (sec)	memory	time (sec)	memory	time (sec)	memory	
128MB	BWT+BOSS	1,486.03		1,501.62		1,496.59		← em-gap
	COSMO	**67.15**	99 MB	**397.53**	6.8 GB	**465.93**	14.4 GB	
512MB	BWT+BOSS	981.49		991.03		991.71		← se-gap
	COSMO	**65.69**	303 MB	**396.76**	6.8 GB	**479.01**	14.4 GB	
	BWT+BOSS*	363.94						← k-gap
2GB	BWT+BOSS	560.95		569.61		564.69		← gap
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BOSS construction

Experiments:[1]

1. Counting k-mers (DSK) and Radix sorting (COSMO) $\leftarrow \mathcal{O}(N \cdot k)$ time.
2. BWT and LCP construction (egap) and BOSS construction $\leftarrow \mathcal{O}(N)$ time.

Results:

- Running time: 22×, 4×
- Peak memory: 1.7× larger, 13.6×

RAM parameter		k=10		k=30		k=50		
		time (sec)	memory	time (sec)	memory	time (sec)	memory	
128MB	BWT+BOSS	1,486.03		1,501.62		1,496.59		← em-gap
	COSMO	**67.15**	99 MB	**397.53**	6.8 GB	**465.93**	14.4 GB	
512MB	BWT+BOSS	981.49		991.03		991.71		← se-gap
	COSMO	**65.69**	303 MB	**396.76**	6.8 GB	**479.01**	14.4 GB	
	BWT+BOSS*	363.94		709.25				← k-gap
2GB	BWT+BOSS	560.95		569.61		564.69		← gap
	COSMO	**78.65**	578 MB	**403.96**	6.8 GB	**479.39**	14.4 GB	

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1. Counting k-mers (DSK) and Radix sorting (COSMO) $\leftarrow \mathcal{O}(N \cdot k)$ time.
2. BWT and LCP construction (egap) and BOSS construction $\leftarrow \mathcal{O}(N)$ time.

Results:

- Running time: 22×, 4× and 3× slower.
- Peak memory: 1.7× larger, 13.6× and 28× smaller.

RAM parameter		k=10		k=30		k=50		
		time (sec)	memory	time (sec)	memory	time (sec)	memory	
128MB	BWT+BOSS	1,486.03		1,501.62		1,496.59		← em-gap
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Outline

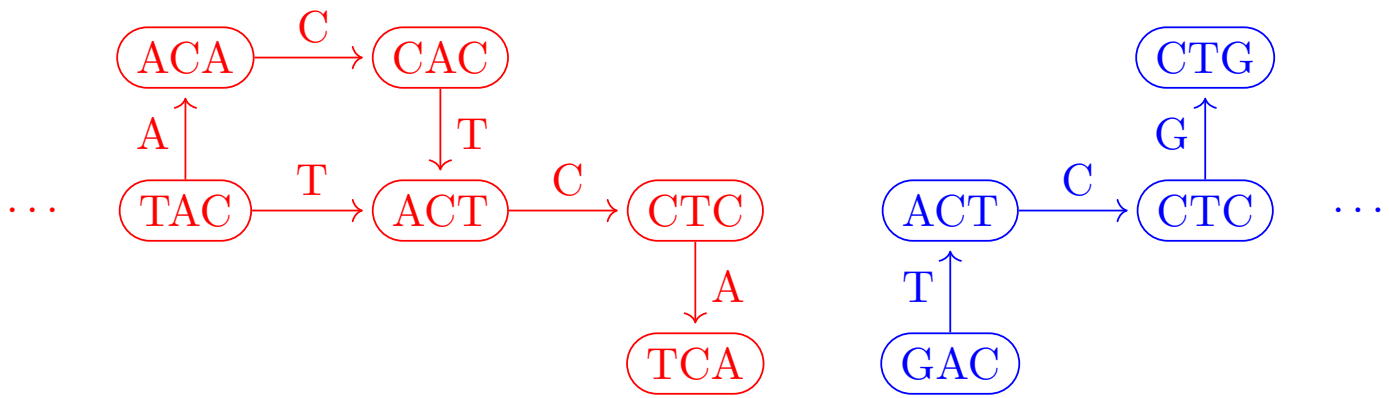
Merging dBGs

Merging BOSS representations

- Suppose we are given the BOSS representation of two* de Brujin graphs $\langle W_0, \text{last}_0 \rangle$ and $\langle W_1, \text{last}_1 \rangle$ from the collections of strings $\mathcal{C}_0$ and $\mathcal{C}_1$
- We compute the BOSS for $\mathcal{C}_{01} = \mathcal{C}_0 \cup \mathcal{C}_1$ directly, that is, without decoding G_0 and G_1 and encode G_{01}.

Example:

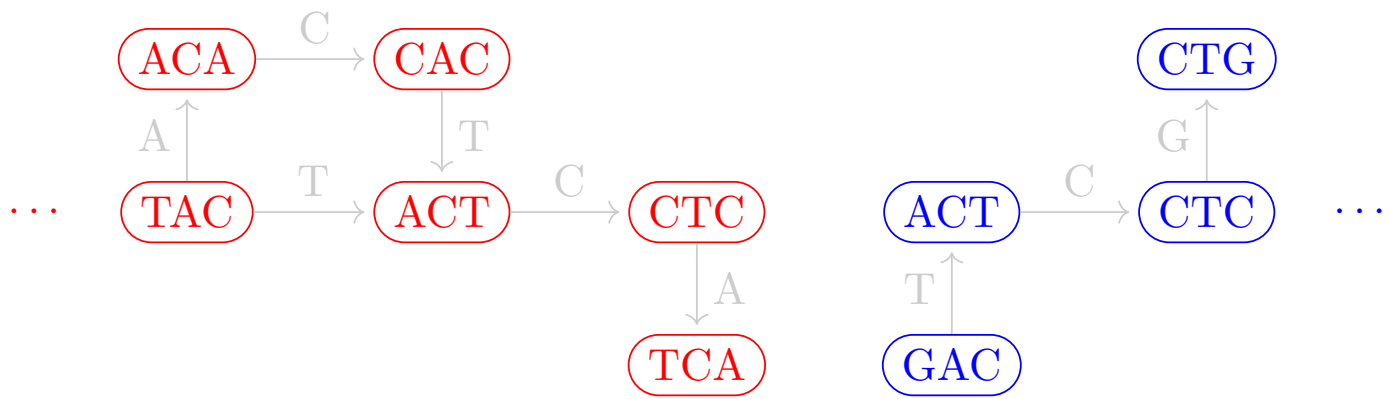
- $\mathcal{S}_1 = \{\$\$\$\text{TACACT}, \$\$\$\text{TACTCA}\} \cup \{\$\$\$\text{GACTCG}\}$



Merging dBGs

Merging BOSS representations

- Tasks:
  1. Merge the nodes in G_0 and G_1 according the order of their k-mers,
     $$\overleftarrow{v_1} \prec \cdots \prec \overleftarrow{v_{n_0}} \qquad \text{and} \qquad \overleftarrow{w_1} \prec \cdots \prec \overleftarrow{w_{n_1}}$$
  2. Recognize when two nodes in G_0 and G_1 refer to the same k-mer, and
     $$\overleftarrow{v_i} \;?=\; \overleftarrow{w_j}$$
  3. Properly merge and update W and last.



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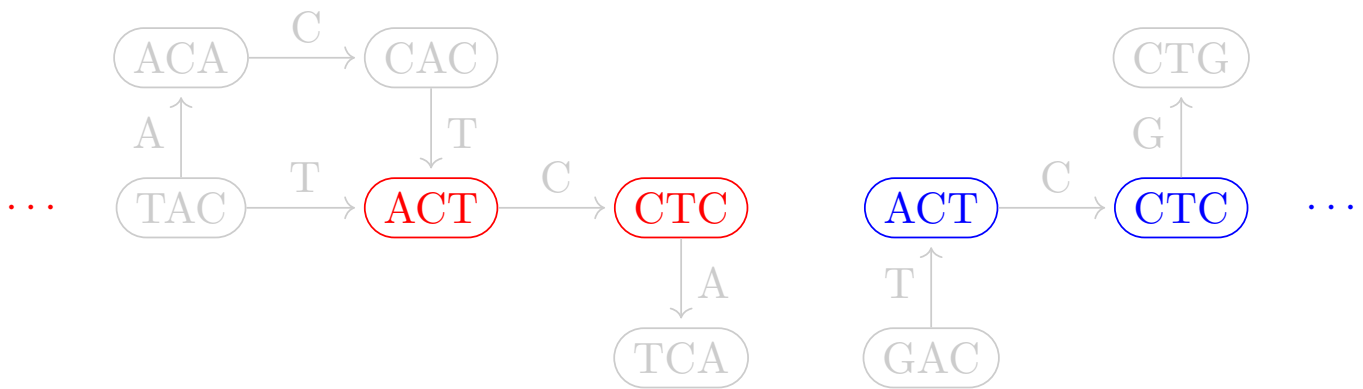
  3. Properly merge and update W and last.



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  1. Merge the nodes in G_0 and G_1 according the order of their k-mers,
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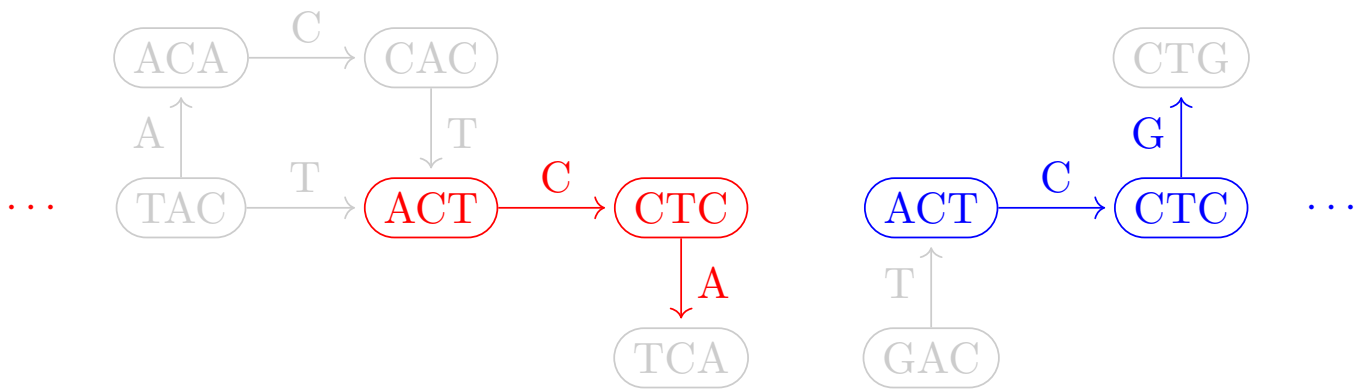
  $$\overleftarrow{v_i} \ ?= \overleftarrow{w_j}$$
  3. Properly merge and update W and last.

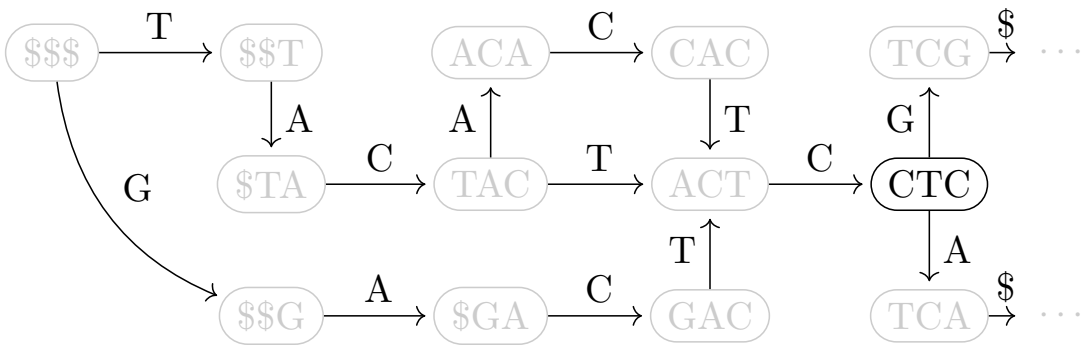


Merging dBGs

1. Merging the nodes in G_0 and G_1:

- The main problem is that in BOSS the k-mers $\overrightarrow{v} = v[1, k]$ are not directly available.
- We will essentially reconstruct them using $\langle W_0, \text{last}_0 \rangle$ and $\langle W_1, \text{last}_1 \rangle$.

last	Nodes	W
0	\$ \$ \$	G
1	\$ \$ \$	T
1	A C A	C
1	T C A	\$
1	\$ G A	C
1	\$ T A	C
1	C A C	T
1	G A C	T-
0	T A C	A
1	T A C	T-
0	C T C	A
1	C T C	G
1	\$ \$ G	A
1	T C G	\$
1	\$ \$ T	A
1	A C T	C



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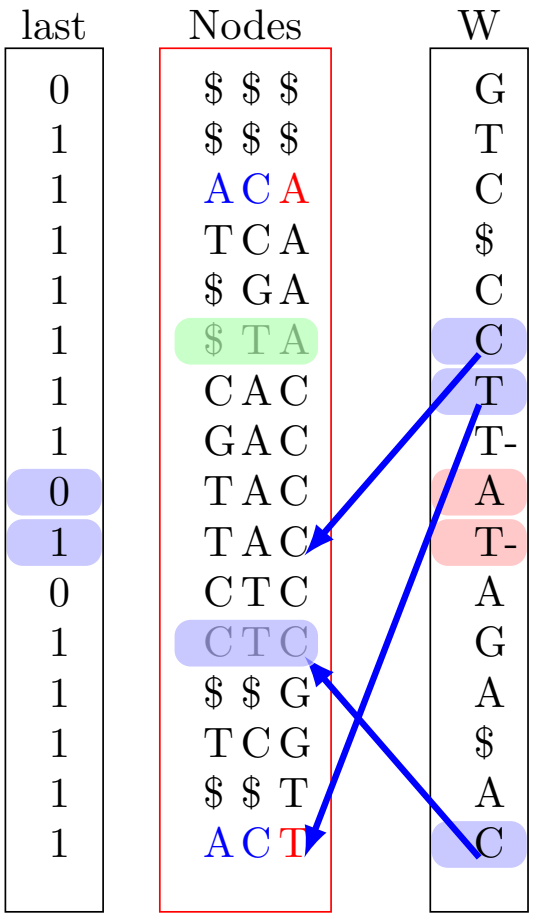

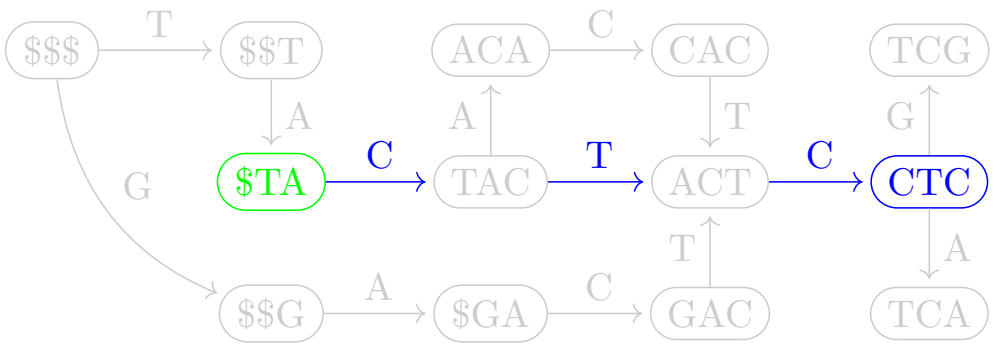

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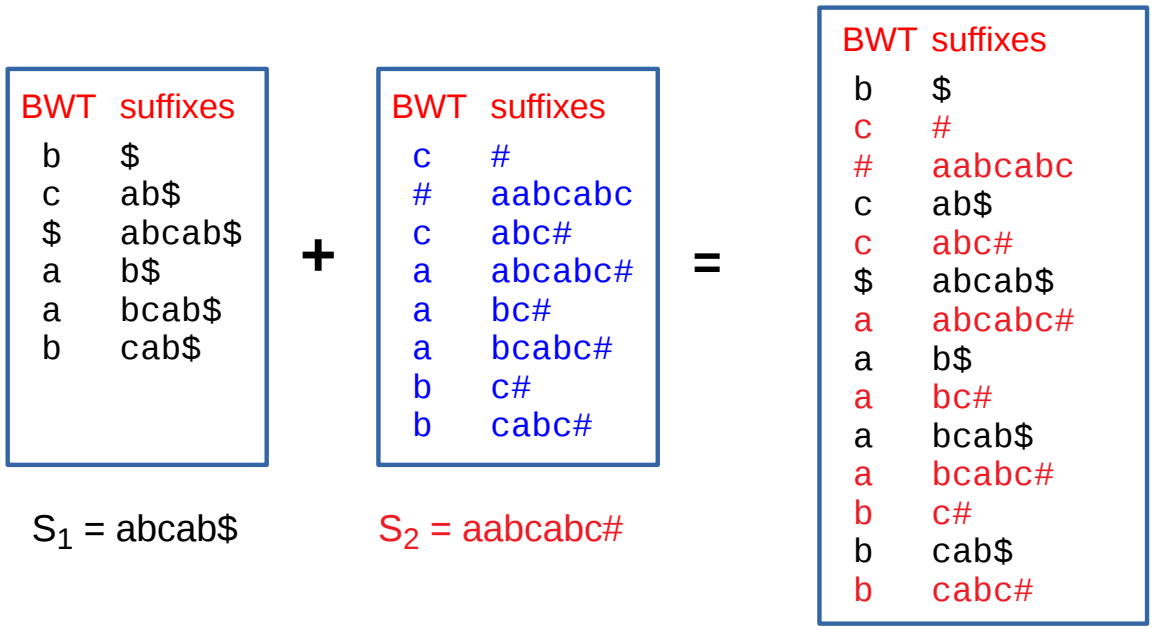
1. Merging the nodes in G_0 and G_1:

- We can merge $\langle W_0, \text{last}_0 \rangle$ and $\langle W_1, \text{last}_1 \rangle$ based on the BWT merging algorithm by Holt and McMillan [Bionformatics 2014, ACM-BCB 2014] .
- Small space: $2n$ bits for Z^{h-1} and Z^h.



1. Radix sort (BWT and bits); 2. At iteration $h = 1, 2, \ldots$, suffixes are h sorted; 3. $\text{BWT}_{01} \leftarrow Z[1, n]$.

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h=1

Z	BWT	
0	b	$
0	c	ab$
0	$	abcab$
0	a	b$
0	a	bcab$
0	b	cab$
1	c	#
1	#	aabcabc
1	c	aabc#
1	a	abcabc#
1	a	bc#
1	a	bcabc#
1	b	c#
1	b	cabc#

h=2

Z	BWT	
0	b	$
1	c	#
1	#	aabcabc
0	c	ab$
0	$	abcab$
1	c	abc#
1	a	abcabc#
0	a	b$
0	a	bcab$
1	a	bc#
1	a	bcabc#
1	b	c#
0	b	cab$
1	b	cabc#

h=3

Z	BWT	
0	b	$
1	c	#
1	#	aabcabc
0	c	ab$
0	$	abcab$
1	c	abc#
1	a	abcabc#
0	a	b$
0	a	bcab$
1	a	bc#
1	a	bcabc#
1	b	c#
0	b	cab$
1	b	cabc#

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h=1 h=2 h=3

Z	BWT	
0	b	$
0	c	ab$
0	$	abcab$
0	a	b$
0	a	bcab$
0	b	cab$
1	c	#
1	#	aabcabc
1	c	aabc#
1	a	abcabc#
1	a	bc#
1	a	bcabc#
1	b	c#
1	b	cabc#

Z	BWT	
0	b	$
1	c	#
0	c	ab$
0	$	abcab$
1	#	aabcabc
1	c	abc#
1	a	abcabc#
0	a	b$
0	a	bcab$
1	a	bc#
1	a	bcabc#
0	b	cab$
1	b	c#
1	b	cabc#

Z	BWT	
0	b	$
1	c	#
1	#	aabcabc
0	c	ab$
0	$	abcab$
1	c	abc#
1	a	abcabc#
0	a	b$
0	a	bcab$
1	a	bc#
1	a	bcabc#
1	b	c#
0	b	cab$
1	b	cabc#

Z	BWT	
0	b	$
1	c	#
1	#	aabcabc
0	c	ab$
0	$	abcab$
1	c	abc#
1	a	abcabc#
0	a	b$
0	a	bcab$
1	a	bc#
1	a	bcabc#
1	b	c#
0	b	cab$
1	b	cabc#

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1	#	aabcabc
1	c	aabc#
1	a	abcabc#
1	a	bc#
1	a	bcabc#
1	b	c#
1	b	cabc#

Z	BWT	
0	b	$
1	c	#
0	c	ab$
0	$	abcab$
1	#	aabcabc
1	c	abc#
1	a	abcabc#
0	a	b$
0	a	bcab$
1	a	bc#
1	a	bcabc#
0	b	cab$
1	b	c#
1	b	cabc#

Z	BWT	
0	b	$
1	c	#
1	#	aabcabc
0	c	ab$
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1	c	abc#
1	a	abcabc#
0	a	b$
0	a	bcab$
1	a	bc#
1	a	bcabc#
1	b	c#
0	b	cab$
1	b	cabc#

Z	BWT	
0	b	$
1	c	#
1	#	aabcabc
0	c	ab$
0	$	abcab$
1	c	abc#
1	a	abcabc#
0	a	b$
0	a	bcab$
1	a	bc#
1	a	bcabc#
1	b	c#
0	b	cab$
1	b	cabc#

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0	a	bcab$
0	b	cab$
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1	#	aabcabc
1	c	aabc#
1	a	abcabc#
1	a	bc#
1	a	bcabc#
1	b	c#
1	b	cabc#

Z	BWT	
0	b	$
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0	a	b$
0	a	bcab$
1	a	bc#
1	a	bcabc#
0	b	cab$
1	b	c#
1	b	cabc#

Z	BWT	
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1	c	#
1	#	aabcabc
0	c	ab$
0	$	abcab$
1	c	abc#
1	a	abcabc#
0	a	b$
0	a	bcab$
1	a	bc#
1	a	bcabc#
1	b	c#
0	b	cab$
1	b	cabc#

Z	BWT	
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1	c	#
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0	c	ab$
0	$	abcab$
1	c	abc#
1	a	abcabc#
0	a	b$
0	a	bcab$
1	a	bc#
1	a	bcabc#
1	b	c#
0	b	cab$
1	b	cabc#

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0	a	b$
0	a	bcab$
0	b	cab$
1	c	#
1	#	aabcabc
1	c	aabc#
1	a	abcabc#
1	a	bc#
1	a	bcabc#
1	b	c#
1	b	cabc#

Z	BWT	
0	b	$
1	c	#
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1	a	bcabc#
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1	b	cabc#

Z	BWT	
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1	a	bc#
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1	b	c#
0	b	cab$
1	b	cabc#

Z	BWT	
0	b	$
1	c	#
1	#	aabcabc
0	c	ab$
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1	c	abc#
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1	a	bc#
1	a	bcabc#
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1	b	c#
1	b	cabc#

Z	BWT	
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1	c	#
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0	c	ab$
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1	c	abc#
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1	b	c#
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1	a	abcabc#
0	a	b$
0	a	bcab$
1	a	bc#
1	a	bcabc#
1	b	c#
0	b	cab$
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1. Radix sort (BWT and bits); 2. At iteration $h = 1, 2, \ldots,$ suffixes are h-sorted; 3. $\text{BWT}_{01} \leftarrow Z[1, n]$;

Merging dBGs

1. Merging the nodes in G_0 and G_1:

- We can merge $\langle W_0, \text{last}_0 \rangle$ and $\langle W_1, \text{last}_1 \rangle$ based on the BWT merging algorithm by Holt and McMillan [Bionformatics 2014, ACM-BCB 2014] .
- Small space: $2n$ bits for Z^{h-1} and Z^h.

h=1

Z	BWT	
0	b	$
0	c	ab$
0	$	abcab$
0	a	b$
0	a	bcab$
0	b	cab$
1	c	#
1	#	aabcabc
1	c	aabc#
1	a	abcabc#
1	a	bc#
1	a	bcabc#
1	b	c#
1	b	cabc#

h=2

Z	BWT	
0	b	$
1	c	#
1	#	aabcabc
0	c	ab$
0	$	abcab$
1	c	abc#
1	a	abcabc#
0	a	b$
0	a	bcab$
1	a	bc#
1	a	bcabc#
1	b	c#
0	b	cab$
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h=3

Z	BWT	
0	b	$
1	c	#
1	#	aabcabc
0	c	ab$
0	$	abcab$
1	c	abc#
1	a	abcabc#
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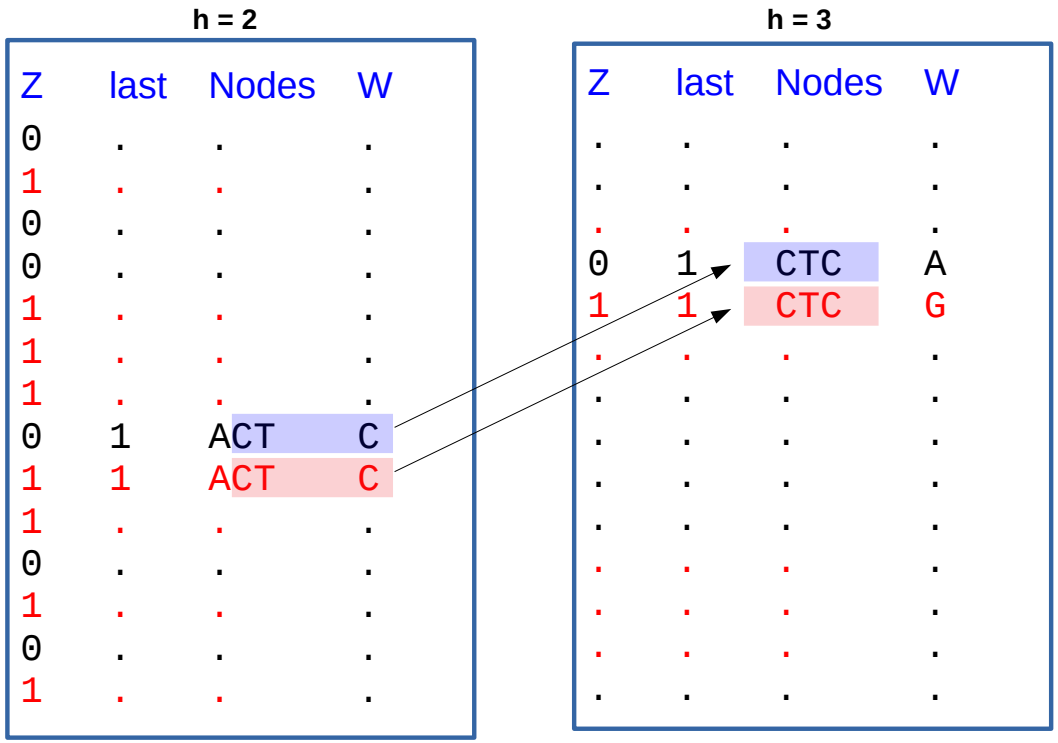
- There are two main modifications:
  1. Proceed by blocks according to array $\text{last}_0[1, n_0]$ (resp. $\text{last}_1[1, n_1]$);
  2. Ignore negative symbols.
- We can also compute the LCP array during HM algorithm [Egidi and Manzini, SPIRE 2017]. ← gap algorithm.



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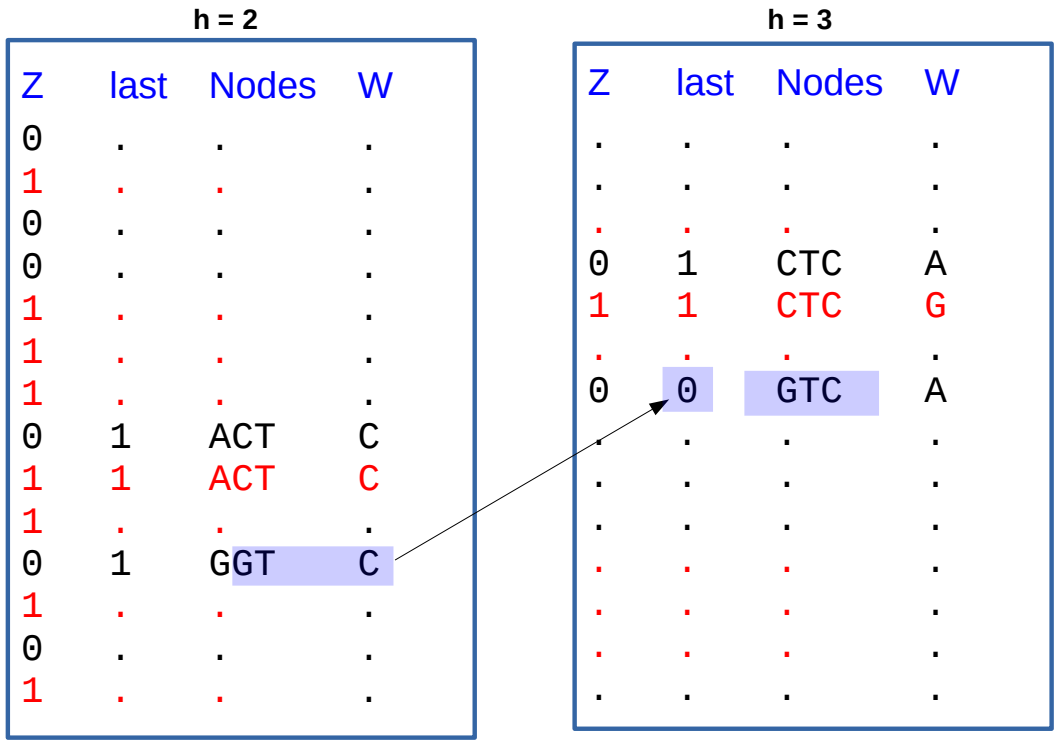

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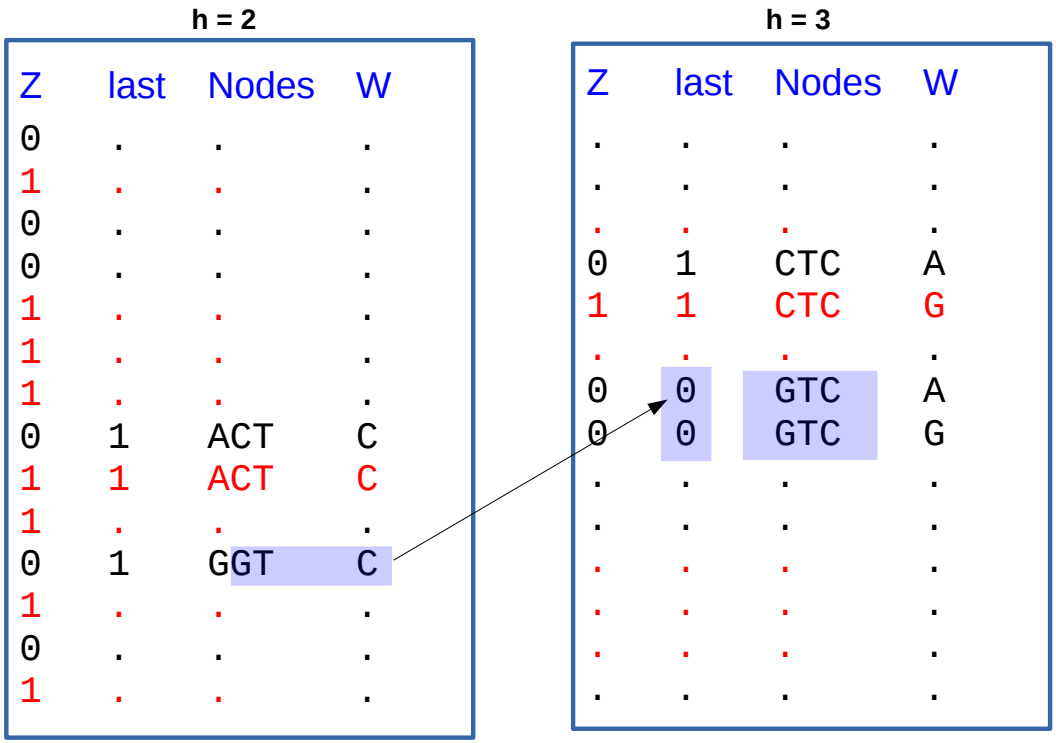

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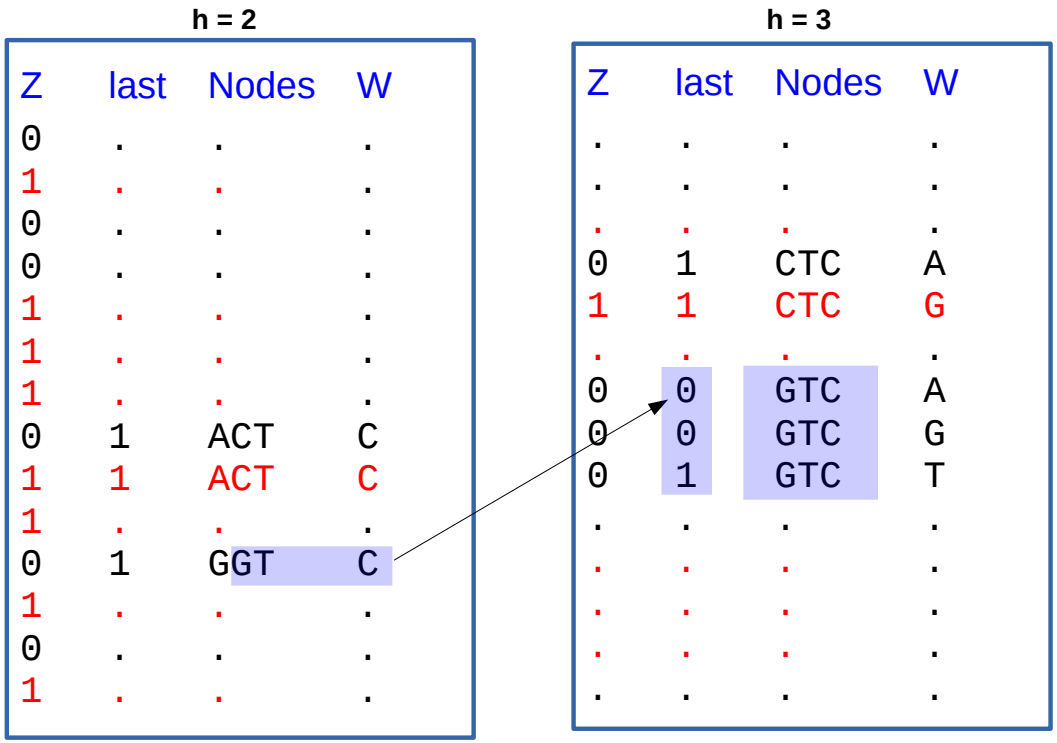

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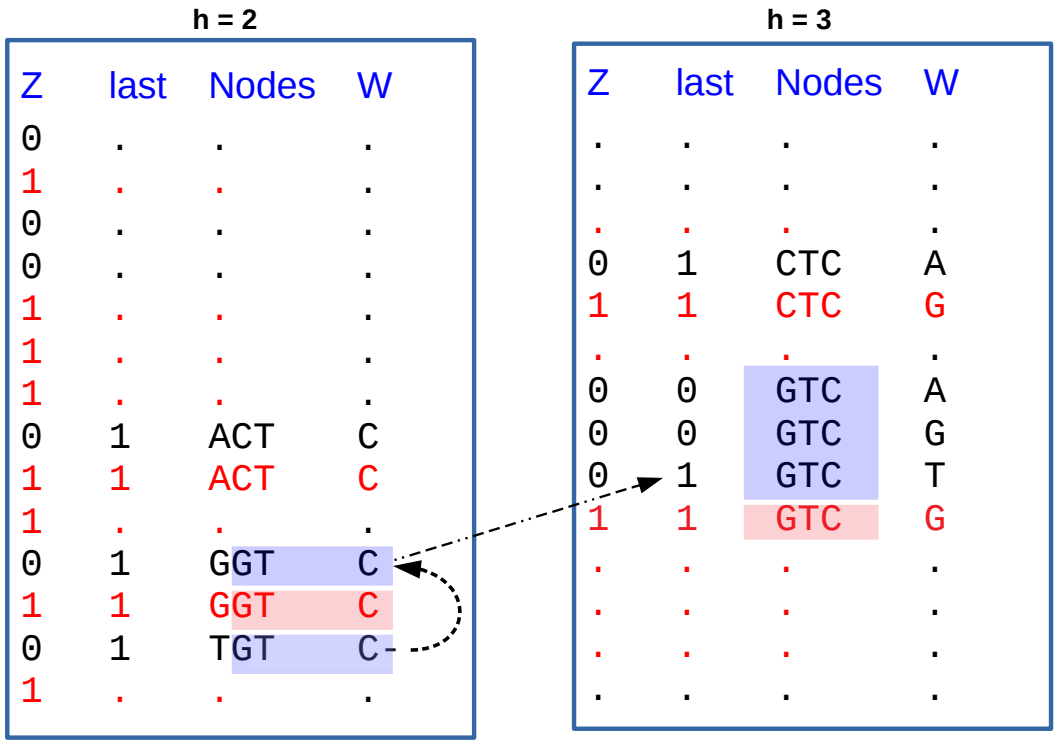

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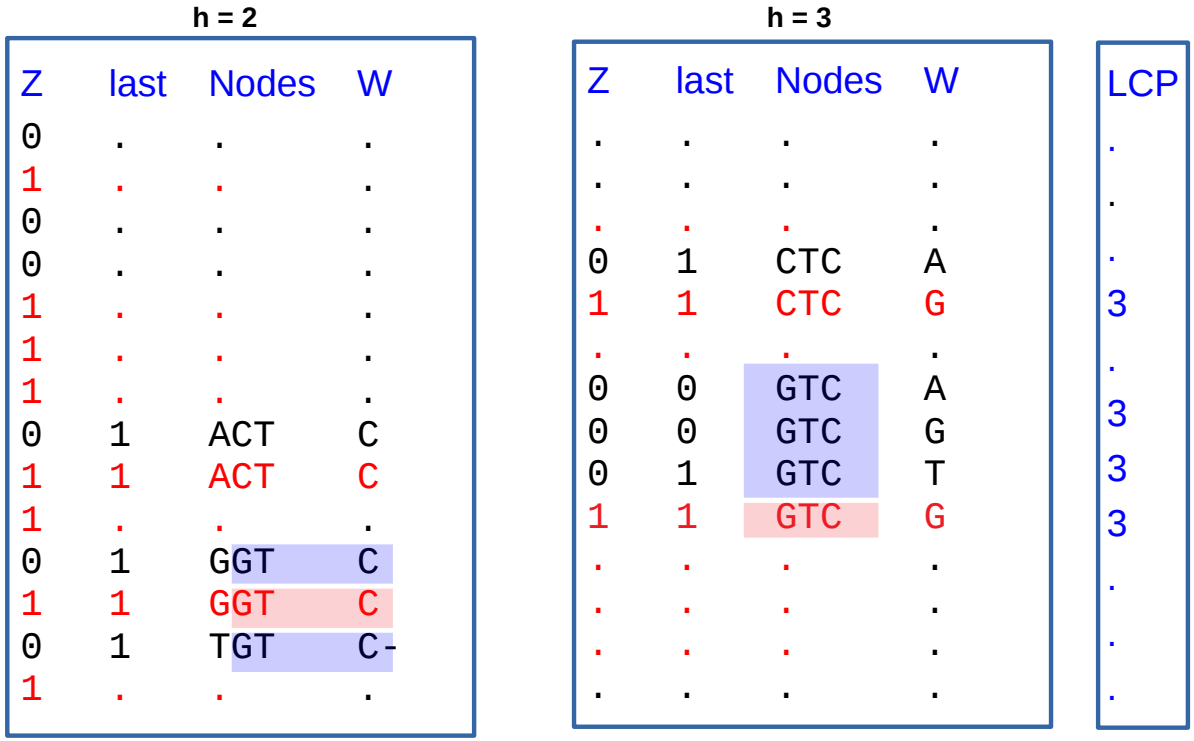


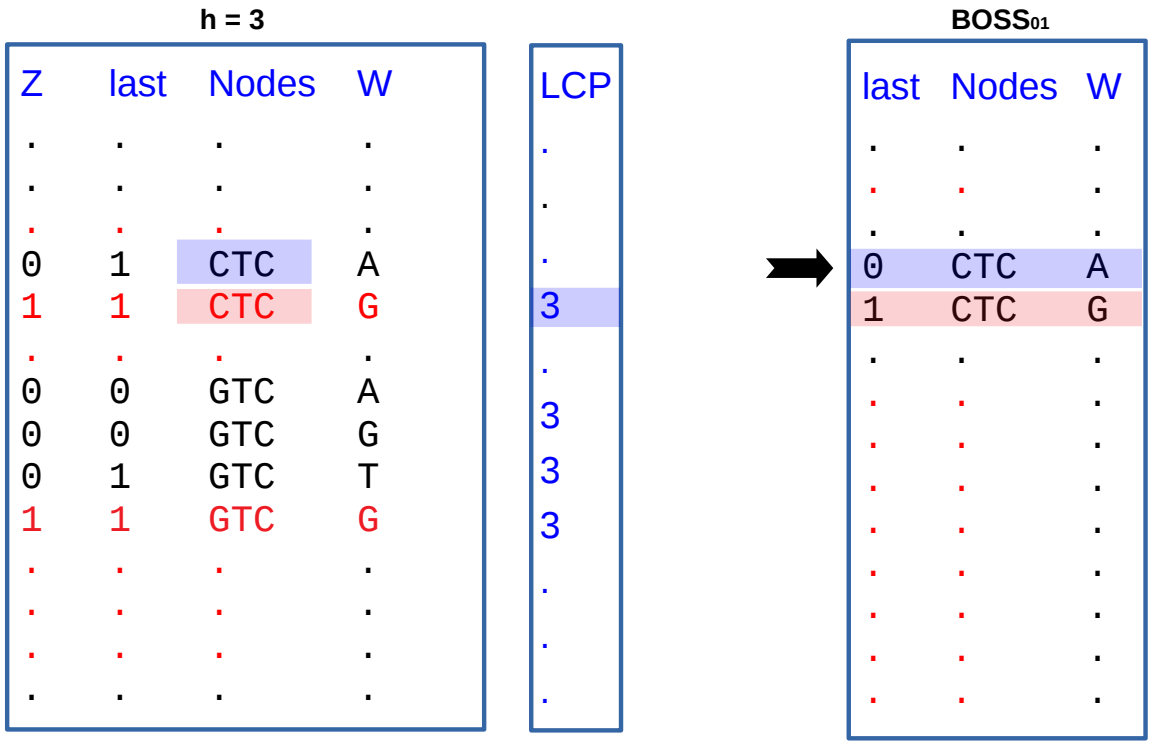
Merging dBGs

2. Recognizing identical k-mers:

- With the LCP array we can decide if $\overleftarrow{v_i}$?= $\overleftarrow{w_j}$.

3. Properly merging $\langle W_0, \text{last}_0 \rangle$ and $\langle W_1, \text{last}_1 \rangle$:

- $Z[1, n]$ and $\text{LCP}[1, n]$ arrays are enough to merge the sorted nodes.
- Update negative symbols, store the last position $\text{LCP}[i] < k - 1$, and for $c \in \Sigma$ store postion of its $pred[c]$.



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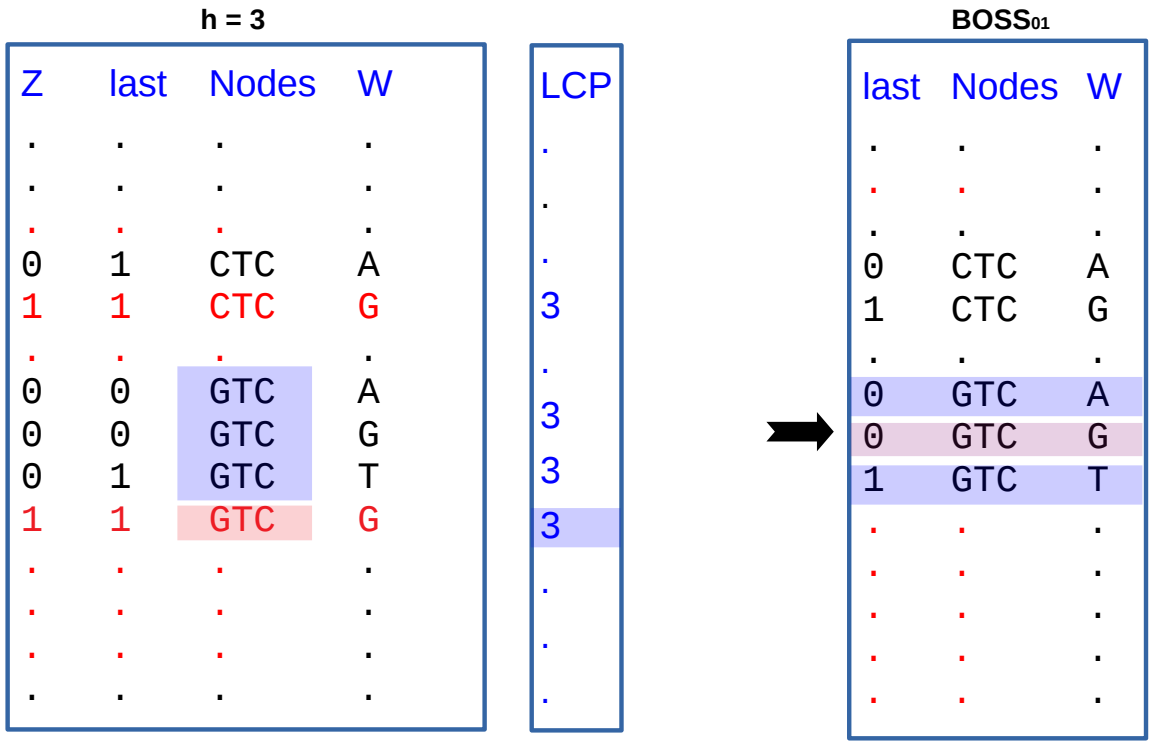

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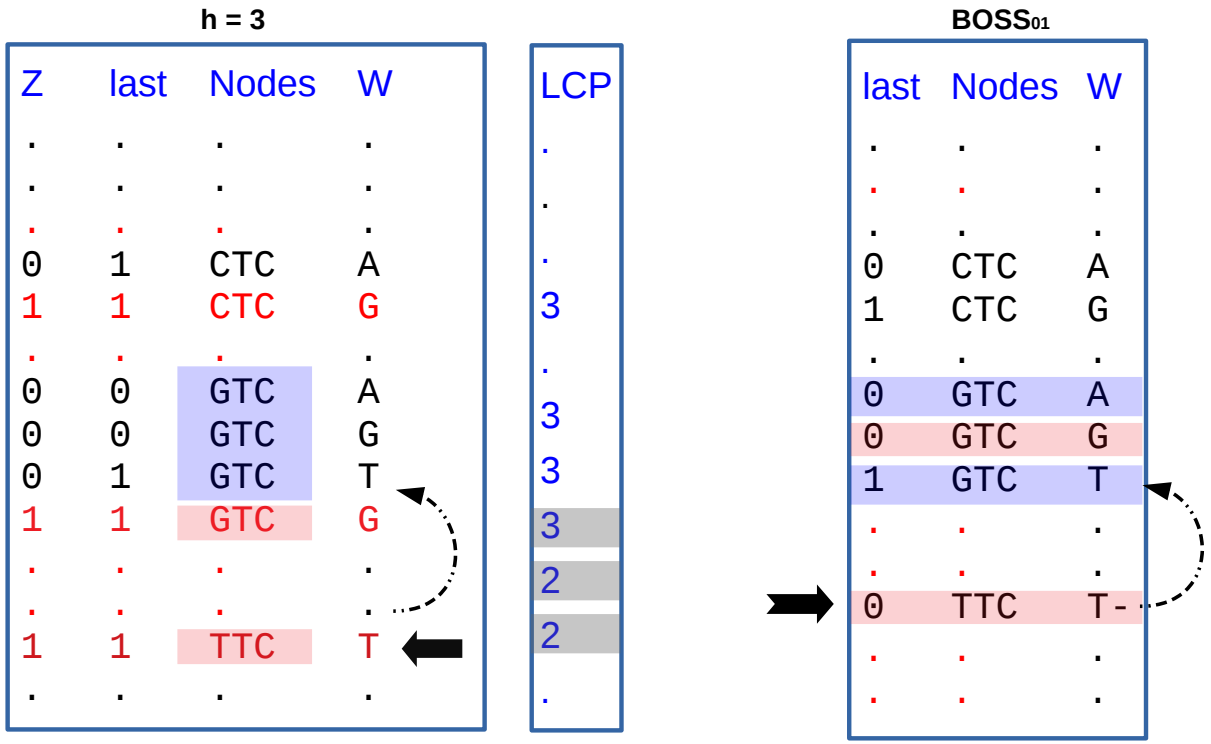

Merging dBGs

Theoretical costs: $m = m_0 + m_1$ (number of edges), and $n = n_0 + n_1$ (number of nodes).

- Running time: $\mathcal{O}(m + n \cdot k)$-time.
  - We use only positive symbols in Ws: we can make a copy of $\langle W_0, \text{last}_0 \rangle$ and $\langle W_1, \text{last}_1 \rangle$ without negative symbols, total $\mathcal{O}(m)$-time.
  - Each step of gap (HM+LCP) algorithm takes $\mathcal{O}(n)$-time, total $\mathcal{O}(n \cdot k)$-time.
- Space: $4n$ bits of working space.
  - $2n$ bits for arrays 2^{k-1} and 2^k.
  - Algorithm gap can encode the LCP array into an array of 2-bit ints, we only need to know

value	code
$\text{LCP}[i] = k$	11
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Merging dBGs

Main result:

The merging of two succinct representations of de Brujin graphs takes $\mathcal{O}(m + n \cdot k)$ time and $4n + \mathcal{O}(1)$ bits of additional space.

Previous result:

Muggli and Boucher [bioRxiv, 2017] showed how to merge de Brujin graphs in $\mathcal{O}(m \cdot k)$ time and $2n(1 + \log \sigma)$ bits of additional space.

Remarks:

- BOSS variations: variable-order and colored.
- Muggli's does not support variable-ordering.
- Most of the data are accessed sequentially ← external memory

1. **Running time** is equivalent for small alphabets, e.g. {A,C,G,T}.
2. **Peak memory** can be prohibitive.

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Outline

Space-efficient construction of succinct de Bruijn graphs

Divide-and-conquer:

1. We split $\mathcal{S}$ into smaller subcollections, ← compute their BWT+LCP array in RAM.
2. For each subcollection: BWT+LCP; then the BOSS representation.
3. Finally, we merge all de Brujin graphs into a single BOSS representation.

Results:

The of de Brujin graph construction for a collection of total length N takes $\mathcal{O}(\log(N/M) \cdot (m + n \cdot k))$ time and $\mathcal{O}(M)$ words and $4n$ bits of RAM.

Remarks:

- We can update (add new graphs) to the de Brujin graph.
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Obrigado!

Felipe A. Louza
louza@usp.br

[arXiv, 2019]

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Outline


Lavinia Egidi, Felipe Alves Louza, Giovanni Manzini, and Guilherme P. Telles.
External memory BWT and LCP computation for sequence collections with applications.
In *WABI*, volume 113 of *LIPIcs*, pages 10:1–10:14. Schloss Dagstuhl - Leibniz-Zentrum fuer Informatik, 2018.


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James Holt and Leonard McMillan.
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Martin D Muggli and Christina Boucher.
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bioRxiv, 2017.