Median of permutations: space reduction techniques and link with the 3-hitting set problem

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* Travaux en collaboration avec Robin Milosz et Adeline Pierrot

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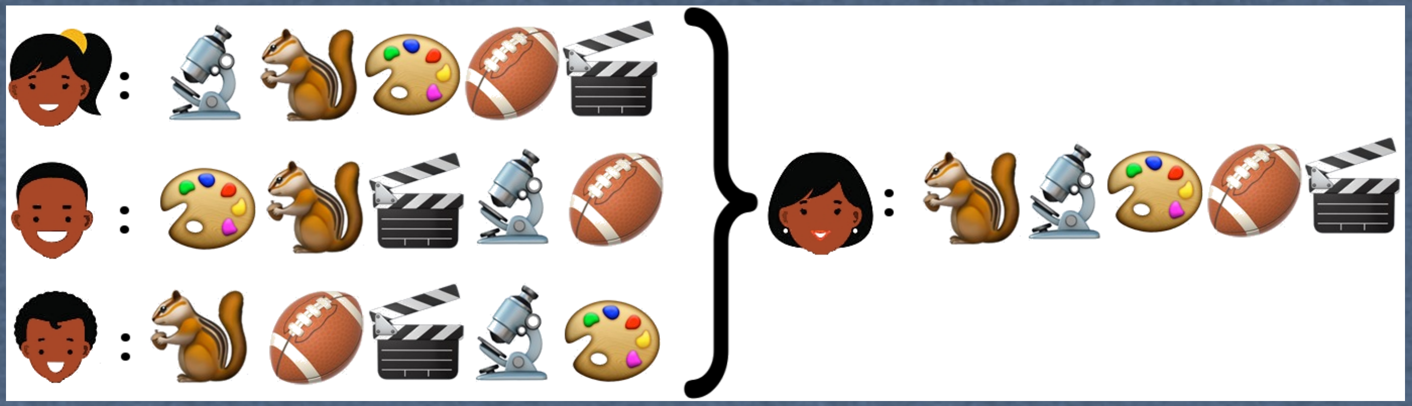

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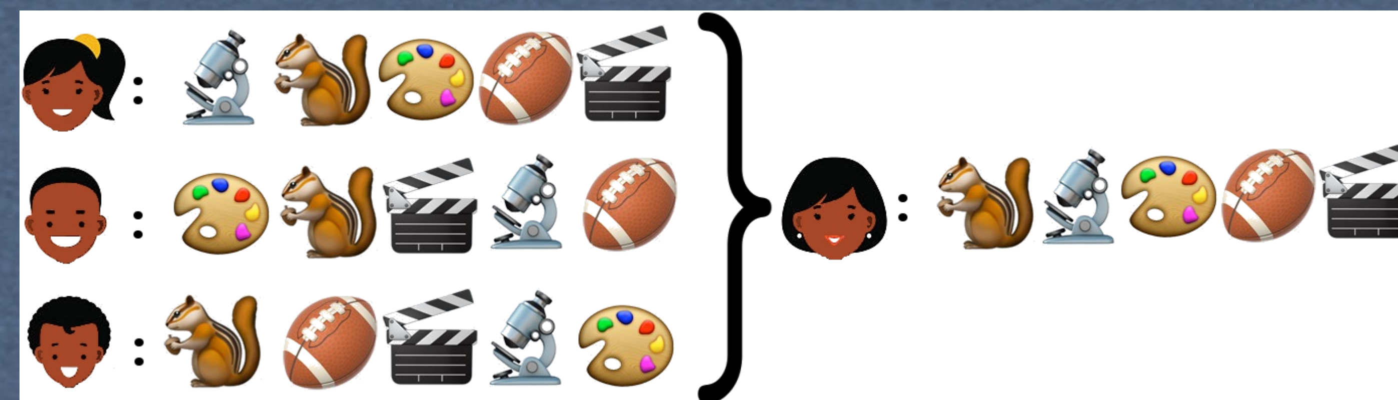
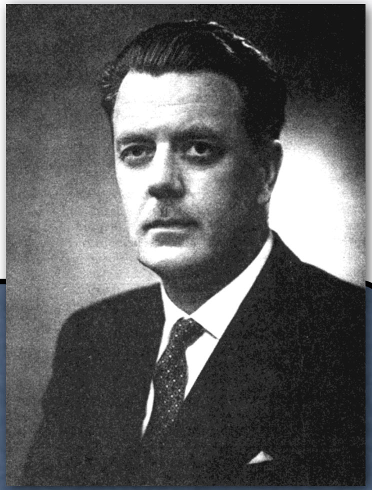


π_1:[4,5,1,2,3]
π_2:[1,5,3,4,2]
π_3:[5,2,3,4,1]

π^*:[5,4,1,2,3]

The Kendall-τ distance:



Maurice Kendall

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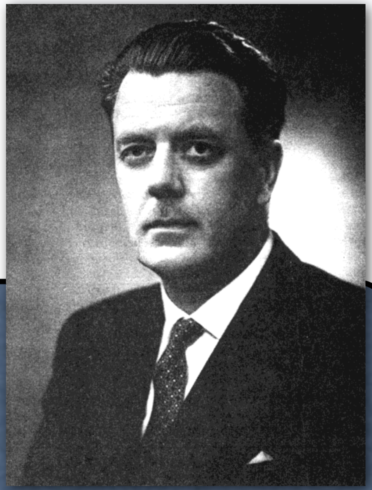

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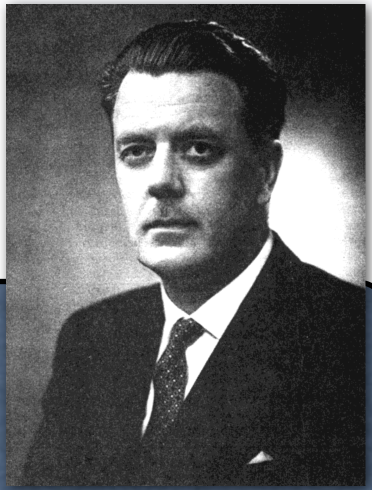

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$$d_{KT}(\pi,\sigma) = \#\{(i,j) | i<j \text{ } and \text{ } [(\pi_i^{-1} < \pi_j^{-1} \text{ } and \text{ } \sigma_i^{-1} > \sigma_j^{-1}) \text{ or } (\pi_i^{-1} > \pi_j^{-1} \text{ } and \text{ } \sigma_i^{-1} < \sigma_j^{-1})]\}$$

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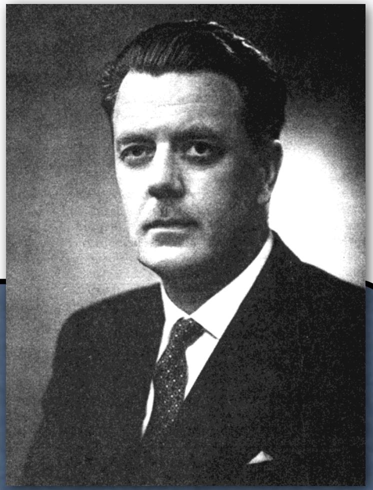
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- The Kendall-τ distance is equivalent to the "bubble-sort" distance i.e. the number of transpositions needed to transform one permutation into the other one.

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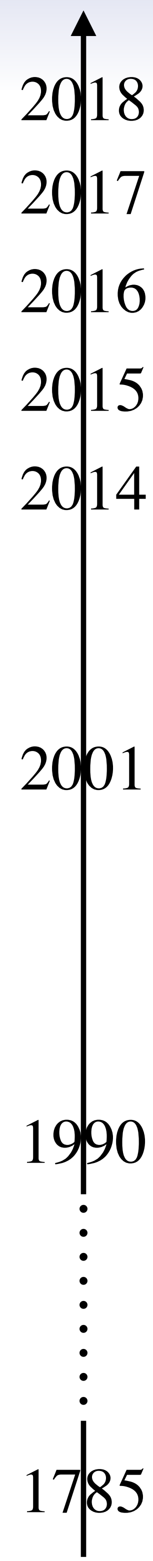
Our problem:

Given a set of m permutations $\mathcal{A} \subseteq \mathcal{S}_n$, we want to find a permutation π^* such that

$$d_{KT}(\pi^*, \mathcal{A}) \leq d_{KT}(\pi, \mathcal{A}), \forall \pi \in \mathcal{S}_n$$

What has been done for space reduction?

Finding a median of a set of m permutations using the Kendall-τ distance



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2001 Dwork *et al.*, NP- hard for even $m \geq 4$

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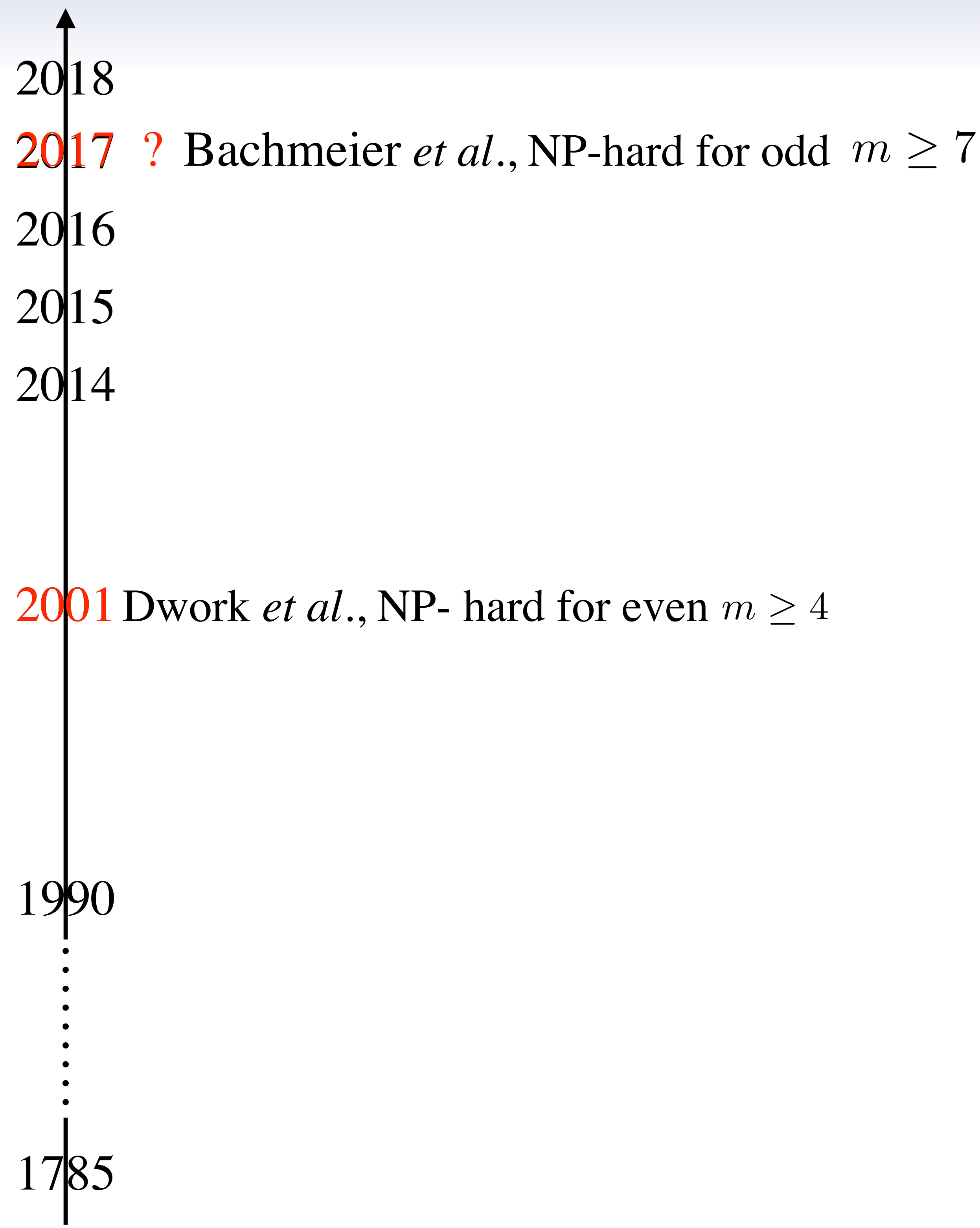

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Pareto criterion or Always Theorem: If a pair of elements appear in the same order in all permutations of the set $\mathcal{A}$, then they also appear in that order in all medians of $\mathcal{A}$.

Betzler *et al*. 2014*: **3/4 majority rule**

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Definition 1: Given a set of permutations $\mathcal{A} \subseteq \mathcal{S}_n$, a non-dirty pair of candidates, according to a certain threshold $s \in [0,1]$, is a pair (a,b), $a,b \in \{1,2,\ldots,n\}$ which respect the following property:

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Theorem: With $s = 0.75$, elements of a median permutation of a set $\mathcal{A}$ will be ordered relatively to a non-dirty candidate in the majority order.

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In other words, a non-dirty candidate will separate the median permutation putting the elements favored to it to its left and the other elements to its right.

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$$\pi^* = [\,\{1,2,4\},3,\{5,6\}\,]$$

* N.Betzler, R.Bredereck, R.Niedermeier, *Theoretical and empirical evaluation of data reduction for exact Kemeny Rank Aggregation*, **Autonomous Agents and Multi-Agent Systems**, vol.28, pp.721-748, 2014.

Applicability of the 3/4 majority rule, in %, on sets of uniformy distributed random permutations. Statistics generated over 10 000 - 400 000 instances:

$m \backslash n$	8	9	10	15	20
3	0.8%	0.55%	0.41%	0.12%	0.05%
4	16.4%	12.88%	10.37%	3.93%	1.92%
5	2.19%	1.57%	1.16%	0.37%	0.18%
6	0.41%	0.28%	0.2%	0.05%	0.02%
7	0.08%	0.05%	0.03%	0.01%	0%
8	0.88%	0.6%	0.43%	0.12%	0.06%
9	0.22%	0.14%	0.09%	0.02%	0.01%
10	0.05%	0.03%	0.02%	0%	0%
15	0%	0%	0%	0%	0%
20	0%	0%	0%	0%	0%

Milsoz *et al*. 2016*: **Major Order Theorems**

Question: Is there a data reduction which is less restrictive than the 3/4 majority rule but more englobing than the always theorem?

* R.Milosz and S.Hamel, *Medians of permutations: building constraints*, **Lecture Notes in Computer Science** 9602, pp. 264-276, 2016.

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- If we limit the interference between two elements, can we derive an extension of the always theorem? YES!

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Example: Let $\mathcal{A} = \{ [7, 8, 2, 3, 6, 1, 5, 4], [3, 5, 1, 7, 8, 6, 2, 4], [5, 8, 3, 4, 1, 2, 7, 6] \}$

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Always

$3 \prec 1$

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Always

$3 \prec 1$

$3 \prec 4$

$3 \prec 6$

$5 \prec 4$

$7 \prec 6$

$8 \prec 2$

$8 \prec 4$

$8 \prec 6$

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$3 \prec 1$

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Always	MOT1
$3 \prec 1$	
$3 \prec 4$	
$3 \prec 6$	
$5 \prec 4$	
$7 \prec 6$	
$8 \prec 2$	
$8 \prec 4$	
$8 \prec 6$	

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Always

$3 \prec 1$
$3 \prec 4$
$3 \prec 6$
$5 \prec 4$
$7 \prec 6$
$8 \prec 2$
$8 \prec 4$
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MOT1

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Always

$3 \prec 1$
$3 \prec 4$
$3 \prec 6$
$5 \prec 4$
$7 \prec 6$
$8 \prec 2$
$8 \prec 4$
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MOT1

$E_{14} = \{2,5,6,7,8\}$

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Always

$3 \prec 1$

$3 \prec 4$

$3 \prec 6$

$5 \prec 4$

$7 \prec 6$

$8 \prec 2$

$8 \prec 4$

$8 \prec 6$

MOT1

$E_{14} = \{2,5,6,7,8\}$

$E_{41} = \{\}$

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Example: Let $\mathcal{A} = \{ [7, 8, 2, 3, 6, \mathbf{1}, 5, \mathbf{4}], [3, 5, \mathbf{1}, 7, 8, 6, 2, \mathbf{4}], [5, 8, 3, \mathbf{4}, \mathbf{1}, 2, 7, 6] \}$

Always

$3 \prec 1$
$3 \prec 4$
$3 \prec 6$
$5 \prec 4$
$7 \prec 6$
$8 \prec 2$
$8 \prec 4$
$8 \prec 6$

MOT1

$E_{14} = \{2,5,6,7,8\}$

$E_{41} = \{\}$

$\delta_{14} = 1 > \#E_{41} = 0$

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Always

$3 \prec 1$

$3 \prec 4$

$3 \prec 6$

$5 \prec 4$

$7 \prec 6$

$8 \prec 2$

$8 \prec 4$

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MOT1

$1 \prec 4$

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Always	MOT1
$3 \prec 1$	$1 \prec 4$
$3 \prec 4$	$1 \prec 6$
$3 \prec 6$	$2 \prec 6$
$5 \prec 4$	$3 \prec 2$
$7 \prec 6$	$5 \prec 1$
$8 \prec 2$	$7 \prec 2$
$8 \prec 4$	
$8 \prec 6$	

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Always	MOT1
$3 \prec 1$	$1 \prec 4$
$3 \prec 4$	$1 \prec 6$
$3 \prec 6$	$2 \prec 6$
$5 \prec 4$	$3 \prec 2$
$7 \prec 6$	$5 \prec 1$
$8 \prec 2$	$7 \prec 2$
$8 \prec 4$	
$8 \prec 6$	

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transitive closure

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Always	MOT1
$3 \prec 1$	$1 \prec 4$
$3 \prec 4$	$1 \prec 6$
$3 \prec 6$	$2 \prec 6$
$5 \prec 4$	$3 \prec 2$
$7 \prec 6$	$5 \prec 1$
$8 \prec 2$	$7 \prec 2$
$8 \prec 4$	
$8 \prec 6$	

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Always	MOT1
$3 \prec 1$	$1 \prec 4$
$3 \prec 4$	$1 \prec 6$
$3 \prec 6$	$2 \prec 6$
$5 \prec 4$	$3 \prec 2$
$7 \prec 6$	$5 \prec 1$
$8 \prec 2$	$7 \prec 2$
$8 \prec 4$	
$8 \prec 6$	

$E_{14} = \{2,5,6,7,8\}$

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Always	MOT1
$3 \prec 1$	$1 \prec 4$
$3 \prec 4$	$1 \prec 6$
$3 \prec 6$	$2 \prec 6$
$5 \prec 4$	$3 \prec 2$
$7 \prec 6$	$5 \prec 1$
$8 \prec 2$	$7 \prec 2$
$8 \prec 4$	transitive closure
$8 \prec 6$	$5 \prec 6$

$E_{14} = \{2,5,6,7,8\}$

$E_{41} = \{\}$

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Always	MOT1	MOT2
$3 \prec 1$	$1 \prec 4$	
$3 \prec 4$	$1 \prec 6$	
$3 \prec 6$	$2 \prec 6$	
$5 \prec 4$	$3 \prec 2$	$E_{14} = \{2,5,6,7,8\}$
$7 \prec 6$	$5 \prec 1$	$E_{41} = \{\}$
$8 \prec 2$	$7 \prec 2$	$\delta_{14} = 1 > \#E_{41} = 0$
$8 \prec 4$	transitive closure	
$8 \prec 6$	$5 \prec 6$	

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Always	MOT1	MOT2
$3 \prec 1$	$1 \prec 4$	
$3 \prec 4$	$1 \prec 6$	
$3 \prec 6$	$2 \prec 6$	
$5 \prec 4$	$3 \prec 2$	$E_{14} = \{2,5,6,7,8\}$
$7 \prec 6$	$5 \prec 1$	$E_{41} = \{\}$
$8 \prec 2$	$7 \prec 2$	$\delta_{14} = 1 > \#E_{41} = 0$
$8 \prec 4$	transitive closure	
$8 \prec 6$	$5 \prec 6$	

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Always

$3 \prec 1$
$3 \prec 4$
$3 \prec 6$
$5 \prec 4$
$7 \prec 6$
$8 \prec 2$
$8 \prec 4$
$8 \prec 6$

MOT1

$1 \prec 4$
$1 \prec 6$
$2 \prec 6$
$3 \prec 2$
$5 \prec 1$
$7 \prec 2$

$E_{14} = \{2,5,6,7,8\}$
$E_{41} = \{\}$
$\delta_{14} = 1 > \#E_{41} = 0$

transitive closure

$5 \prec 6$

MOT2

$E_{24} = \{1,3,5,6\}$

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Example: Let $\mathcal{A} = \{ [7, 8, \mathbf{2}, 3, 6, 1, 5, \mathbf{4}], [3, 5, 1, 7, 8, 6, \mathbf{2}, \mathbf{4}], [5, 8, 3, \mathbf{4}, 1, \mathbf{2}, 7, 6] \}$

Always	MOT1		MOT2
$3 \prec 1$	$1 \prec 4$		
$3 \prec 4$	$1 \prec 6$		$E_{24} = \{1,3,5,6\}$
$3 \prec 6$	$2 \prec 6$		$E_{42} = \{1\}$
$5 \prec 4$	$3 \prec 2$	$E_{14} = \{2,5,6,7,8\}$	
$7 \prec 6$	$5 \prec 1$	$E_{41} = \{\}$	
$8 \prec 2$	$7 \prec 2$	$\delta_{14} = 1 > \#E_{41} = 0$	
$8 \prec 4$	transitive closure		
$8 \prec 6$	$5 \prec 6$		

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Always

$3 \prec 1$

$3 \prec 4$

$3 \prec 6$

$5 \prec 4$

$7 \prec 6$

$8 \prec 2$

$8 \prec 4$

$8 \prec 6$

MOT1

$1 \prec 4$

$1 \prec 6$

$2 \prec 6$

$3 \prec 2$

$5 \prec 1$

$7 \prec 2$

$E_{14} = \{2,5,6,7,8\}$

$E_{41} = \{\}$

$\delta_{14} = 1 > \#E_{41} = 0$

transitive closure

$5 \prec 6$

MOT2

$E_{24} = \{1,3,5,6\}$

$E_{42} = \{1\}$

$\delta_{24} = \delta_{42} = 1$

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Always

$3 \prec 1$
$3 \prec 4$
$3 \prec 6$
$5 \prec 4$
$7 \prec 6$
$8 \prec 2$
$8 \prec 4$
$8 \prec 6$

MOT1

$1 \prec 4$
$1 \prec 6$
$2 \prec 6$
$3 \prec 2$
$5 \prec 1$
$7 \prec 2$

$E_{14} = \{2,5,6,7,8\}$
$E_{41} = \{\}$
$\delta_{14} = 1 > \#E_{41} = 0$

transitive closure

$5 \prec 6$

MOT2

$E_{24} = \{\cancel{1},3,5,6\}$
$E_{42} = \{\cancel{1}\}$
$\delta_{24} = \delta_{42} = 1$

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Always

$3 \prec 1$
$3 \prec 4$
$3 \prec 6$
$5 \prec 4$
$7 \prec 6$
$8 \prec 2$
$8 \prec 4$
$8 \prec 6$

MOT1

$1 \prec 4$
$1 \prec 6$
$2 \prec 6$
$3 \prec 2$
$5 \prec 1$
$7 \prec 2$

$E_{14} = \{2,5,6,7,8\}$

$E_{41} = \{\}$

$\delta_{14} = 1 > \#E_{41} = 0$

transitive closure

$5 \prec 6$

MOT2

$2 \prec 4$

$E_{24} = \{\cancel{1},3,5,6\}$

$E_{42} = \{\cancel{1}\}$

$\delta_{24} = \delta_{42} = 1$

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Always

$3 \prec 1$
$3 \prec 4$
$3 \prec 6$
$5 \prec 4$
$7 \prec 6$
$8 \prec 2$
$8 \prec 4$
$8 \prec 6$

MOT1

$1 \prec 4$
$1 \prec 6$
$2 \prec 6$
$3 \prec 2$
$5 \prec 1$
$7 \prec 2$

$E_{14} = \{2,5,6,7,8\}$

$E_{41} = \{\}$

$\delta_{14} = 1 > \#E_{41} = 0$

transitive closure

$5 \prec 6$

MOT2

$2 \prec 4$
$5 \prec 2$
$7 \prec 4$

$E_{24} = \{\cancel{1},3,5,6\}$

$E_{42} = \{\cancel{1}\}$

$\delta_{24} = \delta_{42} = 1$

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Always

$3 \prec 1$
$3 \prec 4$
$3 \prec 6$
$5 \prec 4$
$7 \prec 6$
$8 \prec 2$
$8 \prec 4$
$8 \prec 6$

MOT1

$1 \prec 4$
$1 \prec 6$
$2 \prec 6$
$3 \prec 2$
$5 \prec 1$
$7 \prec 2$

$E_{14} = \{2,5,6,7,8\}$
$E_{41} = \{\}$
$\delta_{14} = 1 > \#E_{41} = 0$

transitive closure

$5 \prec 6$

MOT2

$2 \prec 4$
$5 \prec 2$
$7 \prec 4$

$E_{24} = \{\cancel{1},3,5,6\}$
$E_{42} = \{\cancel{1}\}$
$\delta_{24} = \delta_{42} = 1$

MOT3

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Always

$3 \prec 1$
$3 \prec 4$
$3 \prec 6$
$5 \prec 4$
$7 \prec 6$
$8 \prec 2$
$8 \prec 4$
$8 \prec 6$

MOT1

$1 \prec 4$
$1 \prec 6$
$2 \prec 6$
$3 \prec 2$
$5 \prec 1$
$7 \prec 2$

$E_{14} = \{2,5,6,7,8\}$

$E_{41} = \{\}$

$\delta_{14} = 1 > \#E_{41} = 0$

transitive closure

$5 \prec 6$

MOT2

$2 \prec 4$
$5 \prec 2$
$7 \prec 4$

$E_{24} = \{\cancel{1},3,5,6\}$

$E_{42} = \{\cancel{1}\}$

$\delta_{24} = \delta_{42} = 1$

MOT3

* R.Milosz and S.Hamel, *Medians of permutations: building constraints*, **Lecture Notes in Computer Science** 9602, pp. 264-276, 2016.

Milsoz *et al*. 2016*: Major Order Theorems

Example: Let $\mathcal{A} = \{ [7, 8, 2, 3, \cancel{6}, 1, 5, 4], [3, 5, 1, 7, 8, \cancel{6}, 2, 4], [5, 8, 3, 4, 1, 2, 7, 6] \}$

Always

$3 \prec 1$
$3 \prec 4$
$3 \prec 6$
$5 \prec 4$
$7 \prec 6$
$8 \prec 2$
$8 \prec 4$
$8 \prec 6$

MOT1

$1 \prec 4$
$1 \prec 6$
$2 \prec 6$
$3 \prec 2$
$5 \prec 1$
$7 \prec 2$

$E_{14} = \{2,5,6,7,8\}$

$E_{41} = \{\}$

$\delta_{14} = 1 > \#E_{41} = 0$

transitive closure

$5 \prec 6$

MOT2

$2 \prec 4$
$5 \prec 2$
$7 \prec 4$

$E_{24} = \{\cancel{1},3,5,6\}$

$E_{42} = \{\cancel{1}\}$

$\delta_{24} = \delta_{42} = 1$

MOT3

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MOT1

$1 \prec 4$
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MOT2

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MOT1

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transitive closure

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MOT2

$2 \prec 4$

$5 \prec 2$

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MOT1

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MOT2

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$5 \prec 2$
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MOT3

$1 \prec 2$
$5 \prec 7$
$6 \prec 4$

21 pairs over 28

* R.Milosz and S.Hamel, *Medians of permutations: building constraints*, **Lecture Notes in Computer Science** 9602, pp. 264-276, 2016.

Milsoz *et al*. 2018*: **Major Order Theorems with equalities**

Dealing with equalities:

* R.Milosz and S.Hamel, *Space reduction constraints for the median of permutations problem*, **Journal of Discrete Applied Mathematics**, in press.

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Dealing with equalities:

- In all of our Major Order Theorems we had

$$\delta_{ij}(\mathcal{A}) > \text{cardinality of interference set}$$

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- What happens when $\delta_{ij}(\mathcal{A})$ = cardinality of interference set ?

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- Can we still use our theorems? YES!

But, we loose the fact that the proven order exist in all medians

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Efficiency on real data:

year	n	m	conflicting pairs	3/4 majority rule	MOT3.0	MOT3.0e
1975	13	14	100%	64.1%	73.1%	100%
1980	19	14	95.9%	84.2%	77.2%	94.8%
1981	19	15	97.7%	73.1%	83.1%	92.4%
1982	9	16	97.2%	100%	86.1%	100%
1983	24	15	98.9%	38.1%	69.2%	76.1%
1984	19	16	99.4%	94.2%	87.1%	96.5%
1985	14	16	100%	93.4%	84.6%	96.7%
1986	21	16	98.6%	92.9%	84.8%	100%
1987	21	16	99.5%	98.6%	82.9%	99.1%
1988	28	16	94.4%	84.1%	89.4%	98.7%
1989	26	16	88.9%	98.2%	88.6%	99.4%
1990	24	16	90.2%	96.4%	90.9%	96.7%
1991	24	16	94.9%	84.8%	84.4%	90.9%
1992	22	16	99.1%	88.3%	84.9%	100%
1993	18	16	98.7%	91.5 %	83.0%	94.1%
1994	16	16	94.2%	95%	70.8%	100%
1995	16	17	100%	97.5%	98.3%	98.3%
1996	19	16	100%	94.8%	84.8%	100%
1997	18	17	100%	83.0%	91.5%	94.8%
1998	21	16	98.1%	97.2%	91.4%	100%
1999	19	16	97.7%	61.4%	74.3%	84.8%
2000	22	17	99.6%	63.7%	87.0%	88.3%
2001	18	17	99.4%	64.1%	78.4%	82.4%
2002	18	17	91.5%	76.5%	87.6%	92.8%
2003	16	16	98.3%	100%	91.7%	100%
2004	15	18	96.2%	100%	92.4%	100%
2005	13	19	100%	96.2%	96.2%	100%
2006	18	18	99.4%	100%	95.4%	100%
2007	18	17	97.4%	91.5%	93.5%	97.4%
2008	20	18	95.8%	81.1%	90%	94.2%

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Comparison of efficiency of MOT 3.0 and MOTe 3.0, in terms of the proportion of ordering of pairs of elements solved, on sets of uniformly distributed random permutations, statistics generated over 100 000 instances for n $\leq$ 80 and 10 000 instances for n=100.

	$n = 15$		$n = 30$		$n = 60$		$n = 100$	
m	MOT 3.0	MOTe 3.0	MOT 3.0	MOTe 3.0	MOT 3.0	MOTe 3.0	MOT 3.0	MOTe 3.0
3	0.635	0.878	0.506	0.701	0.409	0.540	0.356	0.450
4	0.520	0.977	0.413	0.806	0.318	0.506	0.2612	0.369
5	0.581	0.801	0.404	0.595	0.279	0.419	0.219	0.319
10	0.517	0.823	0.361	0.548	0.235	0.349	0.173	0.250
15	0.545	0.704	0.354	0.488	0.225	0.319	0.161	0.227
20	0.525	0.748	0.349	0.492	0.221	0.311	0.157	0.217
25	0.544	0.679	0.350	0.465	0.219	0.300	0.154	0.211
30	0.531	0.718	0.346	0.472	0.216	0.298	0.154	0.208
35	0.547	0.667	0.347	0.454	0.216	0.291	0.152	0.204
40	0.535	0.702	0.345	0.462	0.214	0.291	0.152	0.203
45	0.548	0.660	0.347	0.447	0.214	0.286	0.151	0.200
50	0.537	0.691	0.345	0.455	0.214	0.286	0.150	0.200

Milosz *et al*. 2018*: **3-Cycle Theorem**

Question: What happens if we restrict ourselves to the median of 3 permutations problem, whose complexity is still unknown?

* R. Milosz, S. Hamel et A. Pierrot, *Median of 3 Permutations, 3-Cycles and 3-Hitting Set Problem*, **Lecture Notes in Computer Science** 10979, pp. 224–236, 2018.

Milosz *et al*. 2018*: **3-Cycle Theorem**

Question: What happens if we restrict ourselves to the median of 3 permutations problem, whose complexity is still unknown?

Answer: We can derive an even better data reduction technique with the use of tournament graphs called here majority graphs.

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Majority graph:

$$\left.\begin{aligned}\pi_1 &= [4,5,1,2,3]\\ \pi_2 &= [1,5,3,4,2]\\ \pi_3 &= [5,2,3,4,1]\end{aligned}\right\} \pi^* = [5,4,1,2,3]$$

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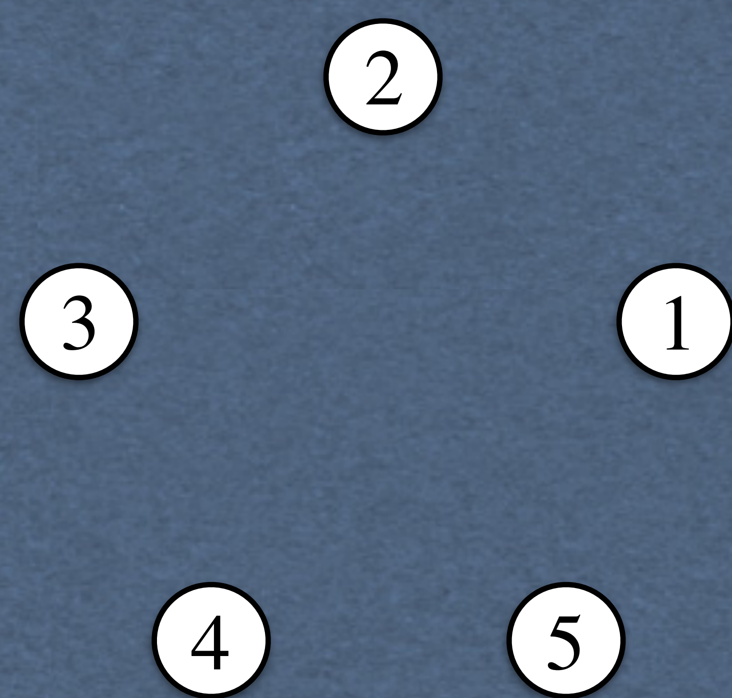

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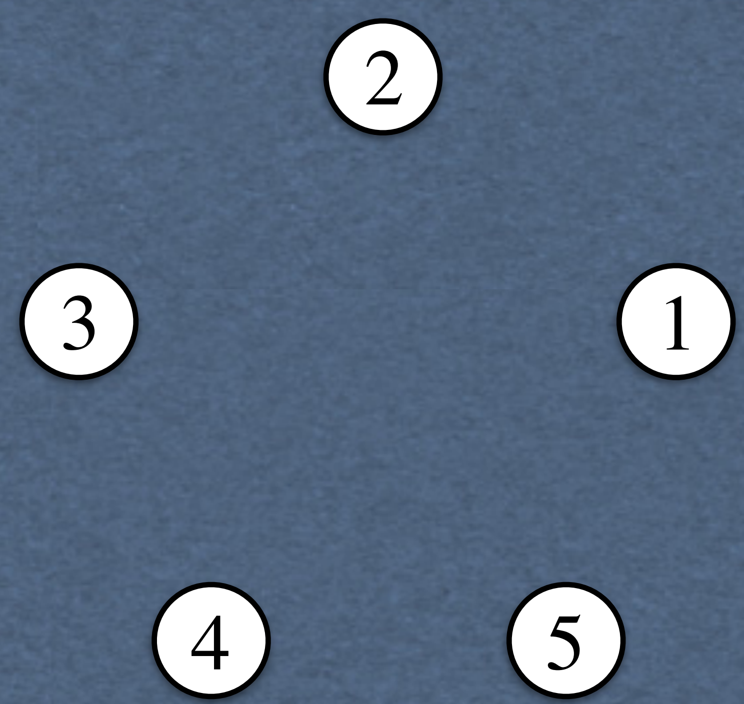

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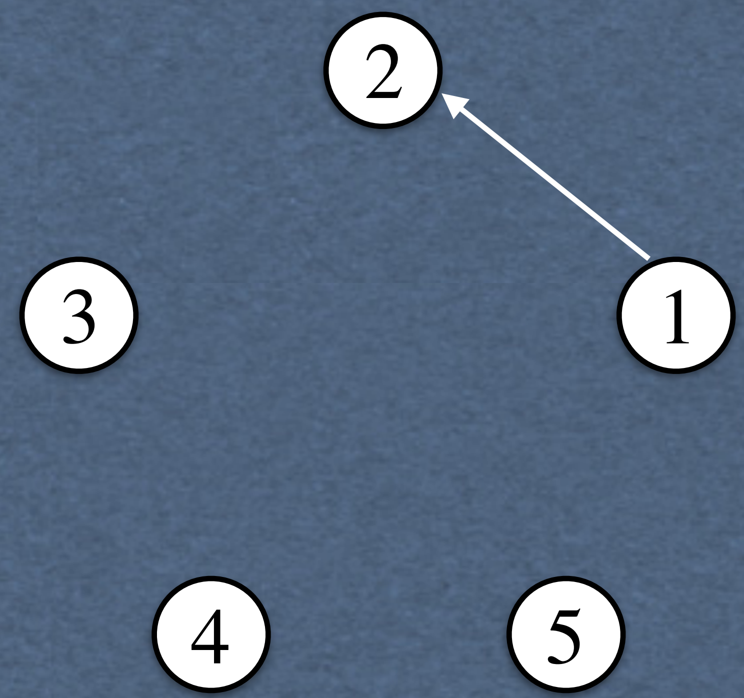

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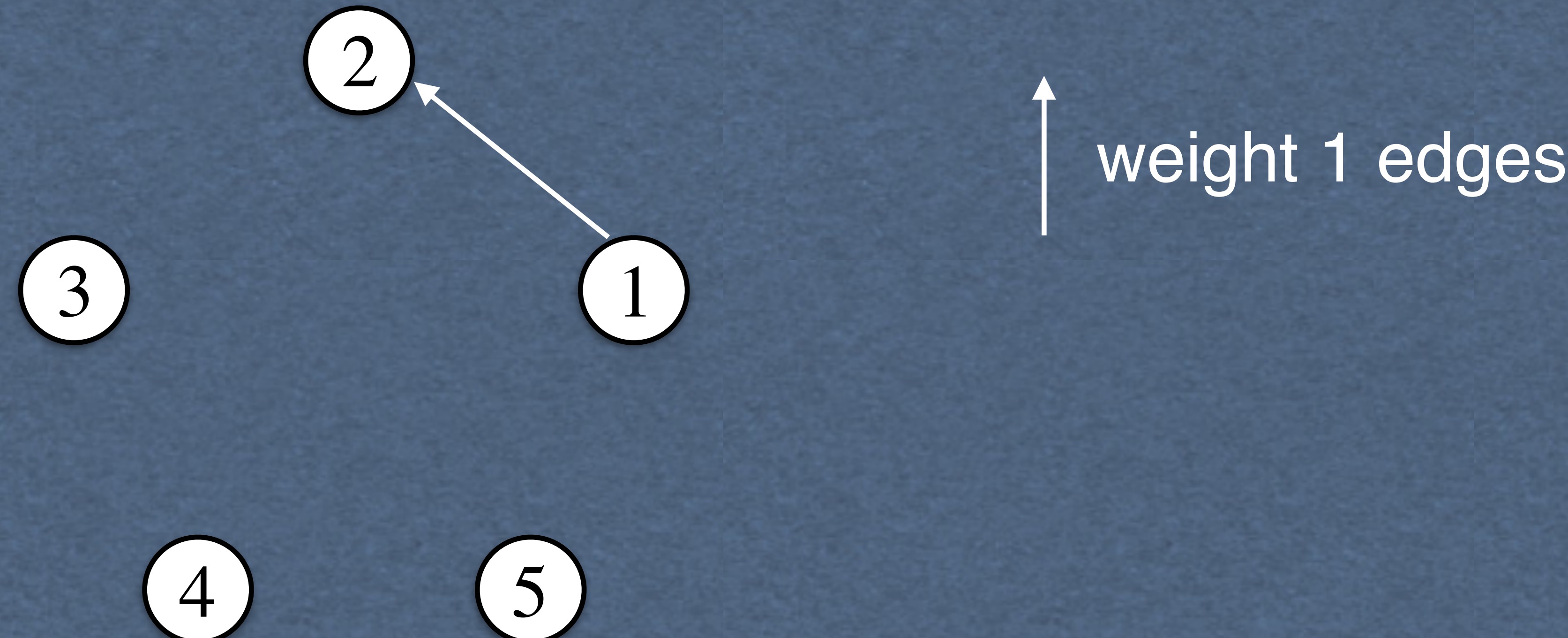

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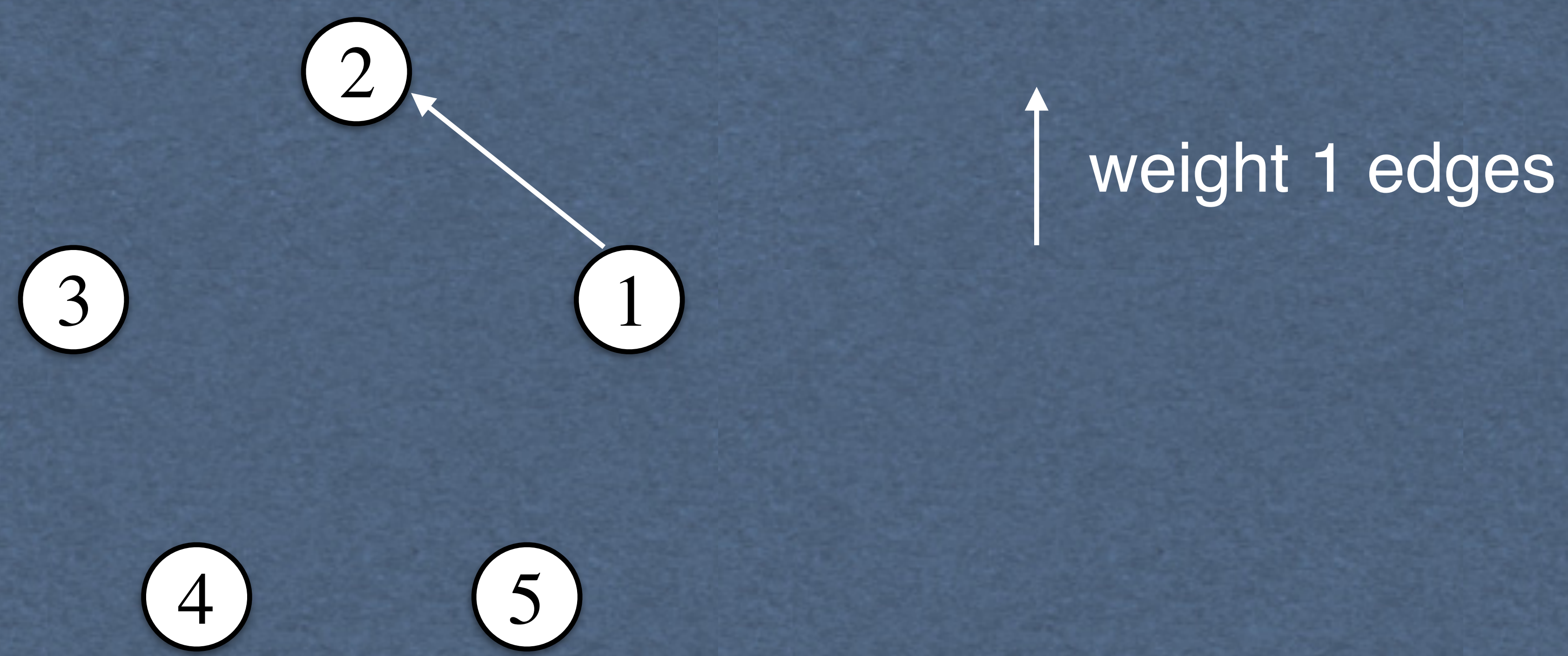

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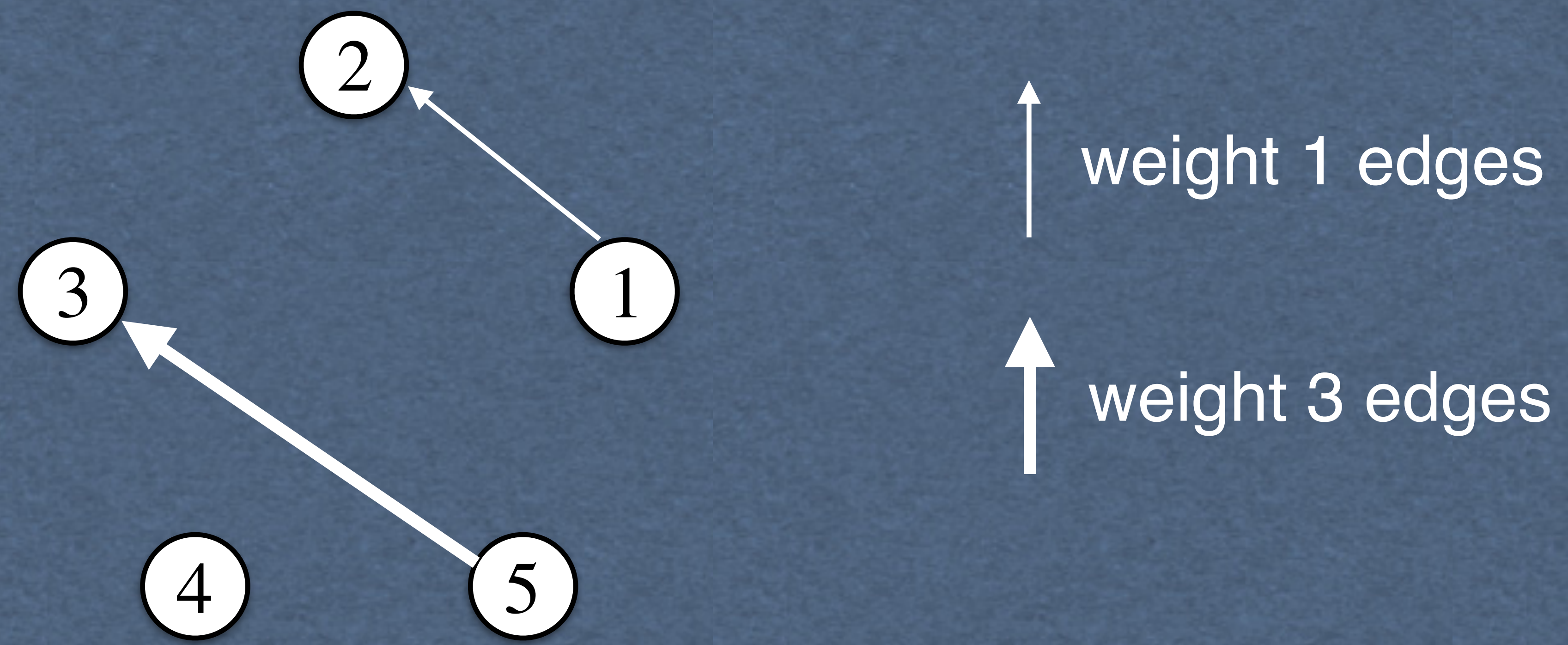

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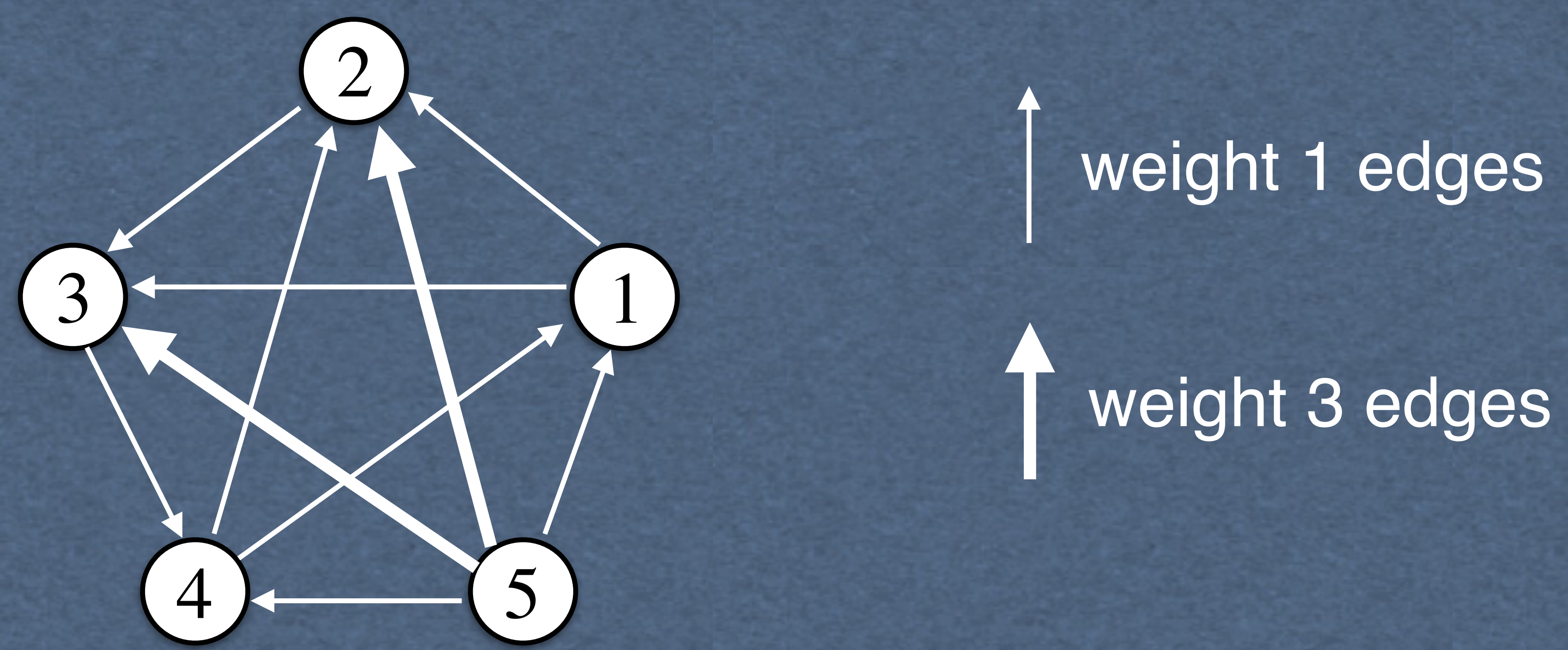

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3-Cycle Theorem: Let $\mathcal{A} \subset \mathcal{S}_n$ be a set of 3 permutations. Let $G_{\mathcal{A}} = (V, E)$ be its majority graph. Let π^* be any median of $\mathcal{A}$. If an edge (i, j) of $G_{\mathcal{A}}$ is not contained in any 3-cycles, then $i \prec_{\pi^*} j$.

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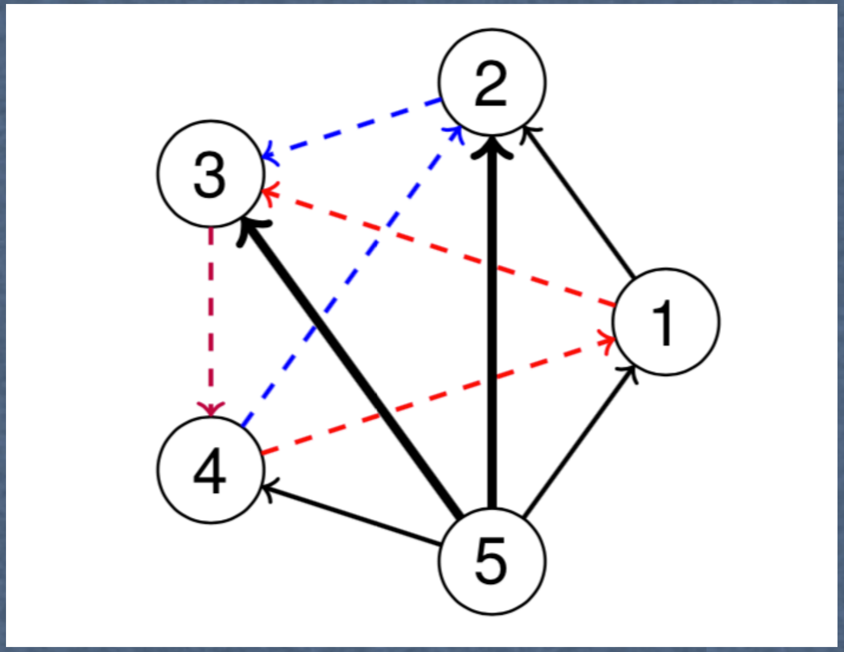

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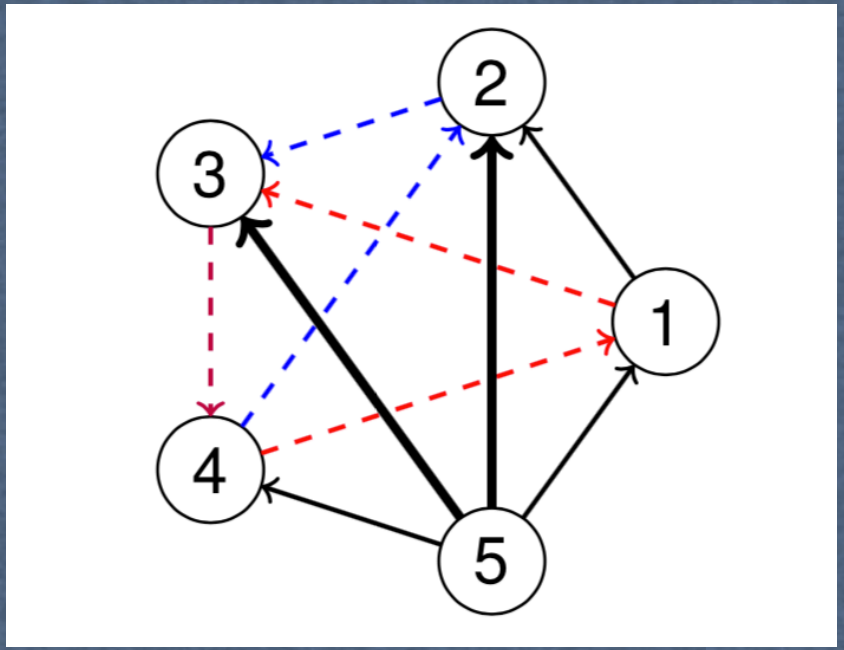

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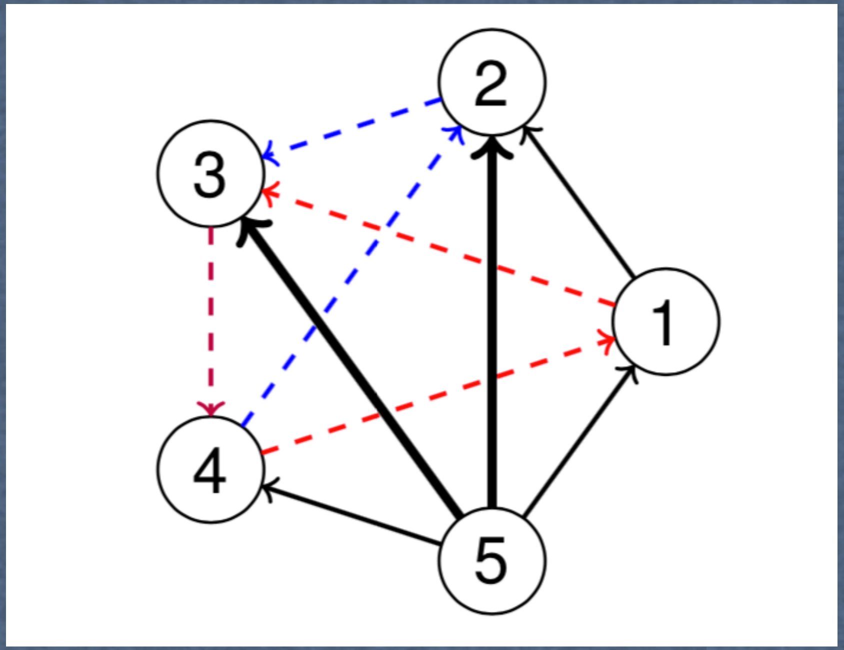

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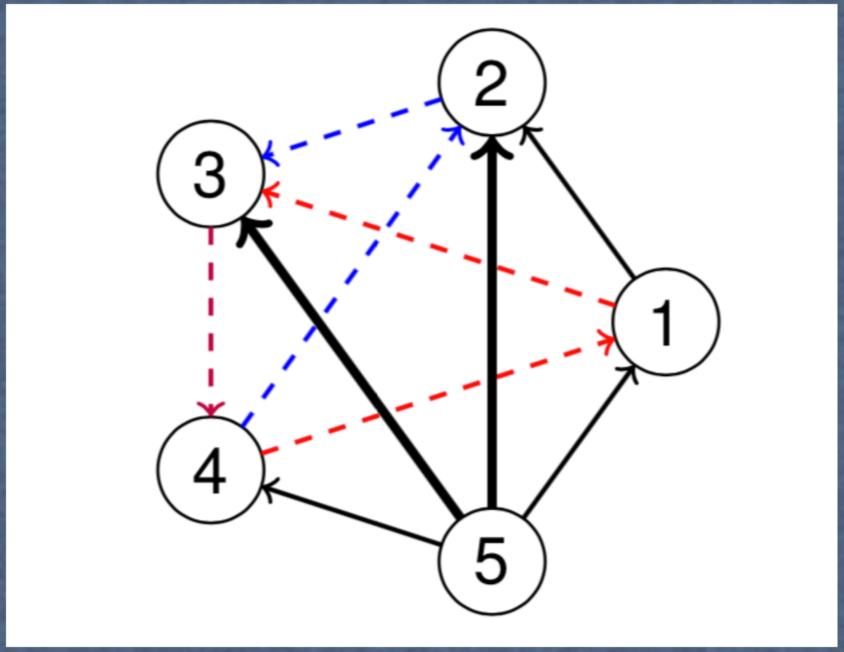

- **Proof:** by contradiction (see article)
- **Reach:** includes and improves all previous space reduction techniques for m=3 permutations
- **Time:** when combined with an ILP solver (CPLEX) it improve the solving time:
1.6X for randomly generated data sets and
3.7X for real life data sets.

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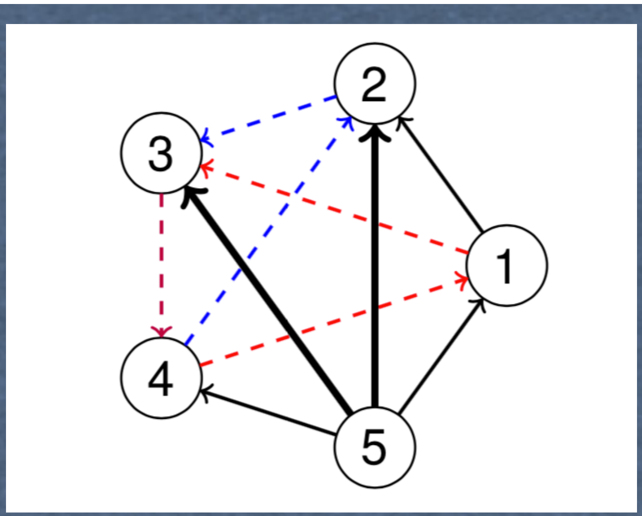

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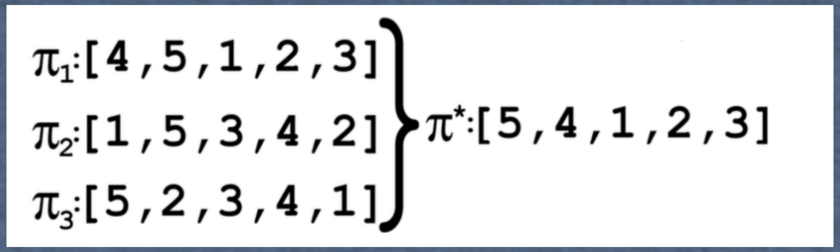

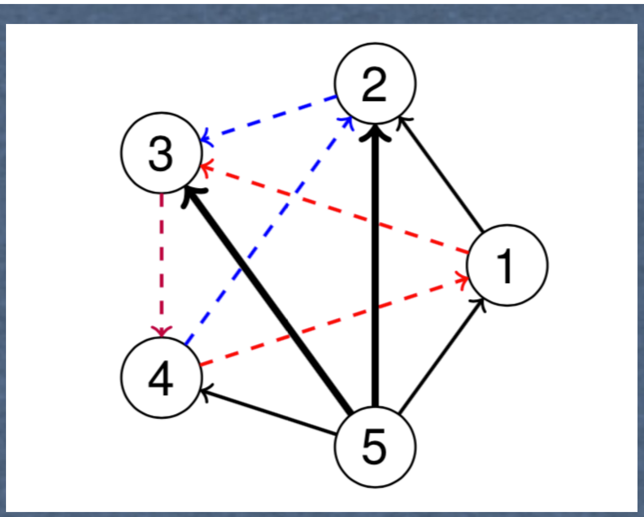

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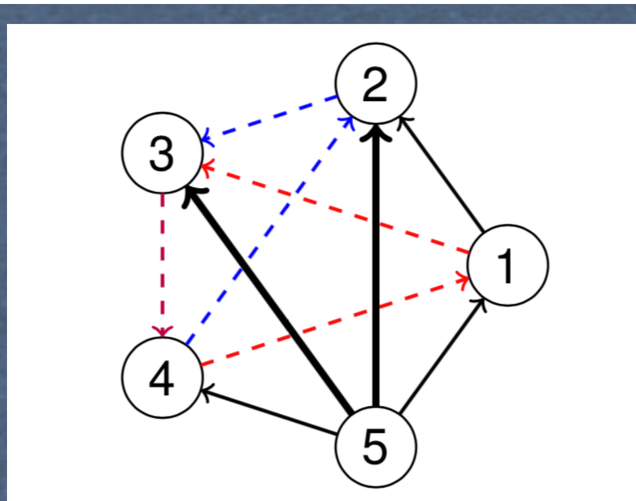

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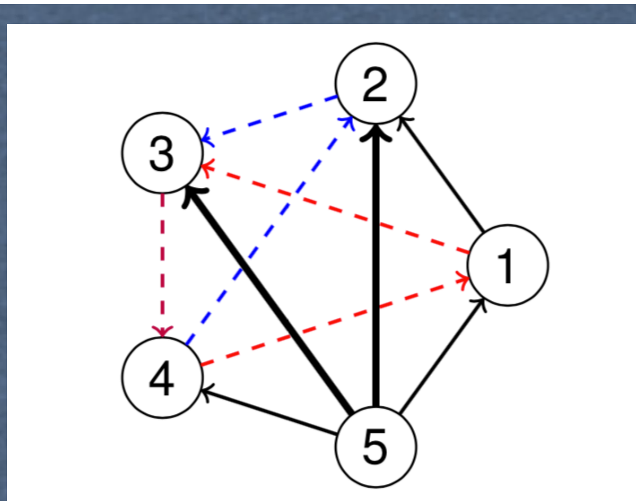

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$e_1 = (2,3)$

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$e_5 = (4,1)$

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$(2,3,4) = t_1$

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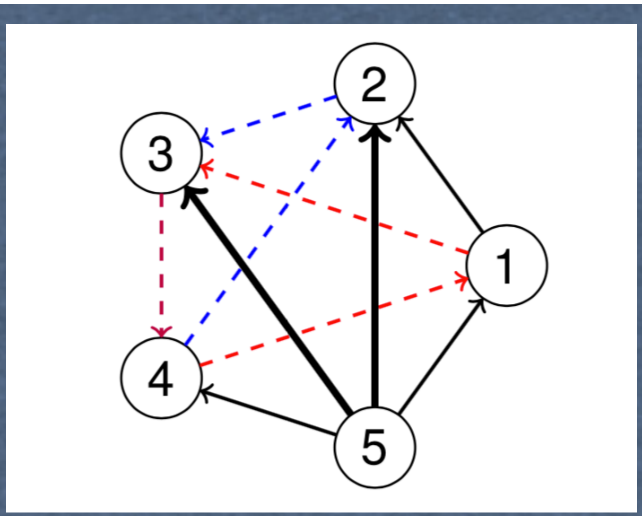

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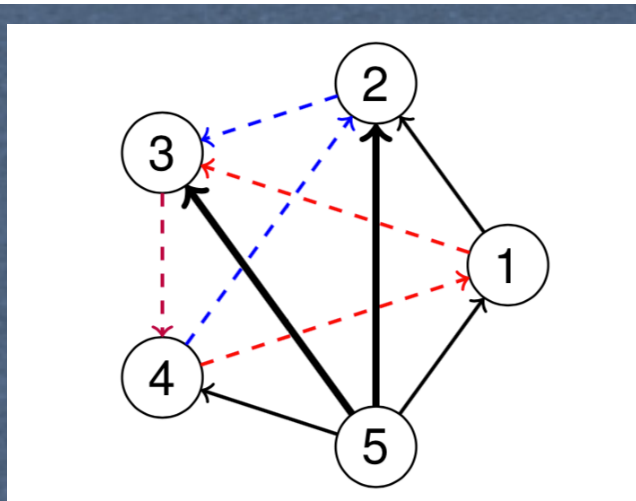

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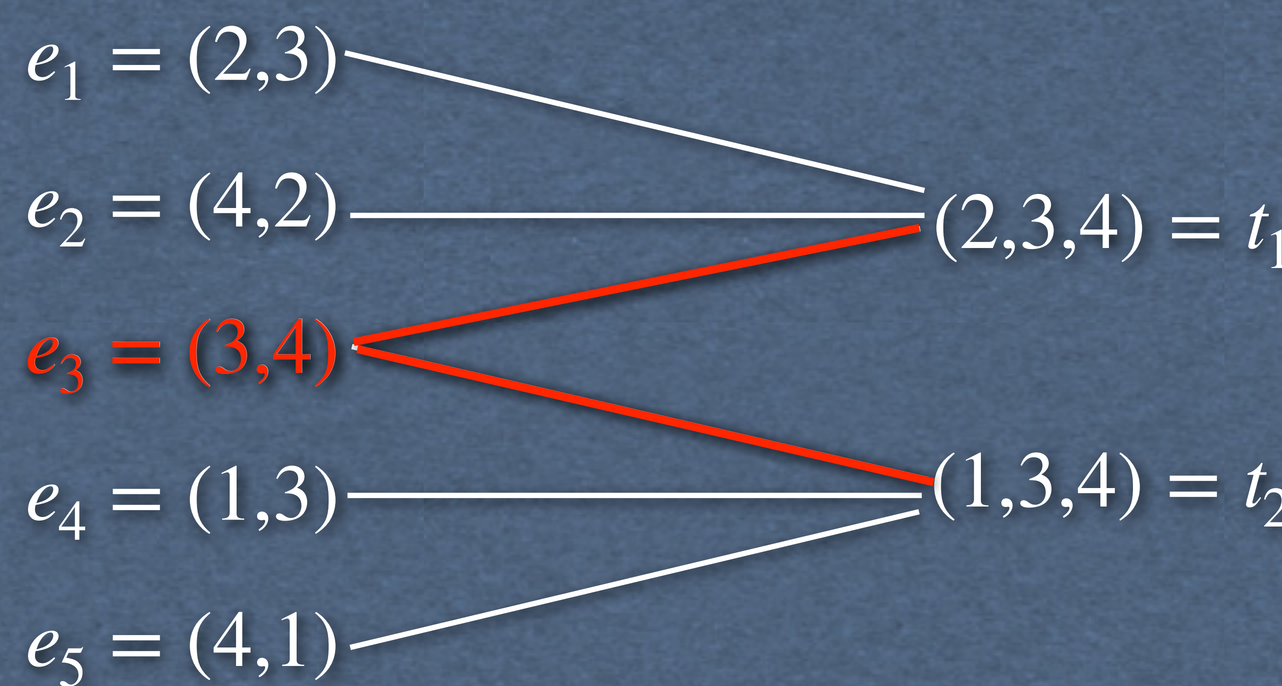

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Conjecture 2: 3-Hitting Set Conjecture for 3 permutations

Let $\mathcal{A} \subset \mathcal{S}_n$ be a set of 3 permutations. Let $G_{\mathcal{A}} = (V, E)$ be its majority graph. Let $\mathscr{E}$ be the set of edges of $G_{\mathcal{A}}$ involve in 3-cycles. Let T be the set of 3-cycles of $G_{\mathcal{A}}$.

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$\Longrightarrow$ ILP solving 19x faster on random data sets and 187x faster on real data sets!!

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- we conjecture that solving the median of 3 permutations problem amounts to solving its corresponding 3 Hitting set problem

Questions

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Real data from PrefLib.org*:

		A	B		C	D	
ID	n	# 3-cycles	max # 3-cycles	A/B	nb constr.	max nb constr.	C/D
ex1	29	78	1015	7.7%	302	406	74.4%
ex2	20	2	330	0.6%	185	190	97.4%
ex3	44	0	3542	0%	946	946	100%
ex4	64	124	10912	1.1%	1848	2016	91.7%
ex5	24	0	572	0%	276	276	100%
ex6	67	1116	12529	8.9%	1457	2211	65.9%
ex7	23	1	506	0.2%	250	253	98.8%
ex8	42	173	3080	5.6%	662	861	76.9%
ex9	28	36	910	4%	326	378	86.2%
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ex14	23	0	506	0%	253	253	100%
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ex16	21	10	385	2.6%	187	210	89%
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ex17	14	0	112	0%	91	91	100%
ex18	23	0	506	0%	253	253	100%
ex19	40	0	2660	0%	780	780	100%
ex20	52	314	5850	5.4%	1046	1326	78.9%

Random uniform data:

		A	B		C	D	
ID	n	# 3-cycles	max # 3-cycles	A/B	nb constr.	max nb constr.	C/D
ex1	29	123	1015	12.1%	260	406	64%
ex2	20	58	330	17.6%	105	190	55.3%
ex3	44	773	3542	21.8%	430	946	45.5%
ex4	64	2356	10912	21.6%	840	2016	41.7%
ex5	24	139	572	24.3%	126	276	45.7%
ex6	67	1839	12529	14.7%	1043	2211	47.2%
ex7	23	97	506	19.2%	140	253	55.3%
ex8	42	566	3080	18.4%	402	861	46.7%
ex9	28	145	910	15.9%	219	378	57.9%
ex10	11	7	55	12.7%	42	55	76.4%
ex11	70	3331	14280	23.3%	824	2415	34.1%
ex12	67	2846	12529	22.7%	833	2211	37.7%
ex13	63	1769	10416	17%	908	1953	46.5%
ex14	23	155	506	30.6%	103	253	40.7%
ex15	43	566	3311	17.1%	449	903	49.7%
ex16	21	100	385	26%	100	210	47.6%
ex17	14	17	112	15.2%	61	91	67%
ex18	23	122	506	24.1%	119	253	47%
ex19	40	507	2660	19.1%	374	780	48%
ex20	52	832	5850	14.2%	689	1326	52%

* N. Mattei et T. Walsh, *Preflib: A library of preference data*, **Lecture Notes in Computer Science** 8176, pp. 259–270, 2013.

Real data from PrefLib.org*:

		A	B		C	D	
ID	n	# 3-cycles	max # 3-cycles	A/B	nb constr.	max nb constr.	C/D
ex1	29	78	1015	7.7%	302	406	74.4%
ex2	20	2	330	0.6%	185	190	97.4%
ex3	44	0	3542	0%	946	946	100%
ex4	64	124	10912	1.1%	1848	2016	91.7%
ex5	24	0	572	0%	276	276	100%
ex6	67	1116	12529	8.9%	1457	2211	65.9%
ex7	23	1	506	0.2%	250	253	98.8%
ex8	42	173	3080	5.6%	662	861	76.9%
ex9	28	36	910	4%	326	378	86.2%
ex10	11	9	55	16.4%	39	55	70.9%
ex11	70	92	14280	0.6%	2283	2415	94.5%
ex12	67	873	12529	6.7%	1524	2211	68.9%
ex13	63	63	10416	0.6%	1856	1953	95%
ex14	23	0	506	0%	253	253	100%
ex15	43	4	3311	0.1%	894	903	99%
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Given a set of m permutations $\mathcal{A} \subseteq \mathcal{S}_n$, we want to find a permutation π^* such that

$$d_{KT}(\pi^*, \mathcal{A}) \leq d_{KT}(\pi, \mathcal{A}), \forall \pi \in \mathcal{S}_n$$

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This median is not always unique

Average number of permutations in *M*(*A*) for uniformly distributed random sets *A of m* permutations of length *n*. Statistics generated over 100 to 1000 instances.

$m \backslash n$	8	10	12	14	15	20	25	30
3	2.1	3.0	3.7	4.8	5.6	12.2	23.1	61.4
4	60.6	331.4	1321.4	7551.4	14253.8	-	-	-
5	2.2	2.9	3.6	5.2	6.2	12.9	29.1	49.2
6	31.3	90.6	345.1	1506.2	1614.9	-	-	-
10	13.0	36.8	88.8	201.9	315.6	2947.9	-	-
15	1.7	2.2	2.8	3.5	3.8	6.3	12.3	-
20	6.3	11.4	22.2	39.8	55.5	256.7	-	-
25	1.6	1.9	2.3	2.6	2.9	4.6	7.6	-

Milsoz *et al*. 2016*: **Major Order Theorems**

Implementation characteristics:

* R.Milosz and S.Hamel, *Medians of permutations: building constraints*, **Lecture Notes in Computer Science** 9602, pp. 264-276, 2016.

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- In practice, $1 \leq k \leq 9$ if $n \leq 400$
- Time for calculating the MOTs is small: < 30 seconds for $n \leq 400$ and $m = 3$

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Applicability of the 3/4 majority rule, in %, on sets of uniformy distributed random permutations. Statistics generated over 10 000 - 400 000 instances:

Inclusion, in %, of the 3/4 majority rule, in Major Order Theorem on the same generated sets

$m \backslash n$	8	9	10	15	20
3	0.8%	0.55%	0.41%	0.12%	0.05%
4	16.4%	12.88%	10.37%	3.93%	1.92%
5	2.19%	1.57%	1.16%	0.37%	0.18%
6	0.41%	0.28%	0.2%	0.05%	0.02%
7	0.08%	0.05%	0.03%	0.01%	0%
8	0.88%	0.6%	0.43%	0.12%	0.06%
9	0.22%	0.14%	0.09%	0.02%	0.01%
10	0.05%	0.03%	0.02%	0%	0%
15	0%	0%	0%	0%	0%
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10	0.05%	0.03%	0.02%	0%	0%
15	0%	0%	0%	0%	0%
20	0%	0%	0%	0%	0%

Inclusion, in %, of the 3/4 majority rule, in Major Order Theorem on the same generated sets

$m\backslash n$	8	9	10	15	20
3	100%	100%	100%	100%	100%
4	85.2%	84.7%	84.0%	86.7%	88.6%
5	100%	100%	100%	99.96%	100%
6	100%	100%	100%	100%	100%
7	100%	100%	100%	100%	100%
8	99.7%	100%	100%	100%	100%
9	100%	100%	100%	100%	100%
10	100%	100%	100%	100%	100%
15	100%	100%	100%	100%	100%
20	100%	100%	100%	100%	100%

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