1. General Background: The Many Faces of ‘Right’

In 1913 Wesley Newcomb Hohfeld set out to clarify the notion of a right.[1] Practicing as a judge, he realized that the word ‘right’ is overused and bringing to the fore the possible meanings behind it would allow us see with clarity what we refer to in the various cases.

The well-known system of correlative pairs of rights and duties he built can be reconstructed in the following diagrams:
First let us examine the group on the left. If I have a parcel of land, I have the privilege (freedom) to walk through it. This means that no one has a claim that I do not go through. This parcel being mine means that others have the duty to stay away: I have the claim-right towards them to do so. As the diagram indicates, claim-right and duty always go together. The same is true for the correlative pair of privilege and no-claim. As the latter label shows: no-claim is the opposite of having a claim-right. We have the freedom to do something exactly when we don’t have a duty to refrain from it; more exactly—as Hohfeld emphasizes: "always, when it is said that a given privilege is the mere negation of a duty, what is meant, of course, is a duty having a content or tenor precisely the opposite to that of the privilege in question." [1]

The group on the right side exhibits a very similar structure; there are various points of difference between the groups though. Hohfeld already laid down that in the rights (powers) in the second group are above claim-rights in a sense: with power-rights (but not with claim-rights) one does (not) have the possibility to change legal positions; for instance: if I have a parcel, I have the power to sell it, but selling it changes my claim-rights and privileges connected to it. Therefore the members of the second group are usually considered higher-order rights. Among others, Fitch[2] and Makinson[3] argued for considerations along which the difference seems to be more sophisticated. Fitch emphasized that the expressed modalities’ nature is different: while we can call the members of the claim-right group deontic modalities, the group of power involves some kind of capacity, so we should call them capacitative modalities. Makinson adds that meanwhile if we do something without a permission, we have to expect a sanction, if we do something without power, we actually do not do it. These observations are crucial when formalizing the notions.

From the viewpoint of deontic logic the virtue of the Hohfeldian system is that it handles agency—this is the point of correlativity: someone’s right always involves someone else’s duty, and the other way around. These agents in pairs can be called counterparts. Rights and duties do not exist on their own: being someone’s right and someone else’s duty inheres in their essence. This correlativity is crucial: it helps us decide whether a right/duty exists on the basis of the correlative duty/right’s existence, possibility, acceptability... So this is a point of great importance on which I build my proposal providing formal definitions on the Hohfeldian conceptions.
2. Major Underlying Considerations in Formalization

2.1. Counterparty
ess

While the notion of counterparties has been found fundamental in the reception of Hohfeld, it is one of the main points Marek Sergot considered as shortcomings of the most notable formalization done by Kanger and Kanger (1966), later Kanger (in 1971, 1972, 1985), and Lindahl (1977, 1994). As Sergot[5] writes in his article presenting and discussing this work: "the theory of normative position, when viewed as a theory of duties and rights or as a formalisation of the Hohfeldian framework fails to deal with the notion of counterparty". Presenting an attempt to build upon this notion he says "counterparty as a claimant notion is associated with 'power'" and cites Makinson[3] giving a definition for obligation using the notion of counterparty and the notion of power:

\[ x \text{ bears an obligation to } y \text{ that } F \text{ under the system } N \text{ of norms iff in the case that } F \text{ is not true then } y \text{ has the power under the code } N \text{ to initiate legal action against } x \text{ for non-fulfillment of } F \]

By this suggestion, considering the correlation and the directedness of the obligation, Makinson practically describes the claim-right of \( y \) also. The obvious difficulty with this—intuitive-sounding—definition (as Sergot points out also) is the right-left direction of the biconditional: \( y \) can initiate a legal action against anyone without having a claim-right. It’s just that he won’t win the case. This is why it would be a natural idea to involve some kind of expectation of success in the definition, a point Sergot raises.

However, an expectation of success in itself doesn’t reveal the nature of the duty or its correlative, the claim-right. It shows some correlation in a statistical sense but nothing more. In my formalization I would like to demonstrate how this expectation comes about and what it means considering the type of the claim-right as a right (and a duty as an obligation).

Makinson’s definition above building on counterparties points to another (if not the most) important attribution of legal rights from a legal theoretical viewpoint: the possibility of seeking remedy in court.

2.2. First Group’s Formal Representation: Enforceability by the State

The possibility of seeking remedy in court is a well-grounded expectation in western legal culture: the Declaration of the Rights of Man and Citizen (Approved by the National Assembly of France, August 26, 1789), after listing the rights, declares in the paragraph before the last: “A society in which the observance of the law is not assured (...) has no constitution at all.” Considering this issue a bit more generally: among norms, the possibility of seeking remedy in court is the differentia specifica of legal ones. It might seem intuitive to regard sanction as the hallmark of a legal norm, but would not be true: for Catholic people, going to Hell does sound as a sanction in the case of religious norms; also, ostracism could be really unpleasing which is a reason behind following social norms. But none of these norms has the State behind. And while rights and, especially, duties as notions can be associated with morality and ethics also, Hohfeld himself was
a judge and called these notions as fundamental legal conceptions. Therefore it seems natural, reasonable, and justified to capture their essence by building on their specificity making them legal rights and duties, just like Makinson did. In my proposal, though, not power, but claim-right plays the crucial role in describing the possibility of seeking remedy in court.

2.3. Clarifying the Role of the State

The correlative nature of rights and duties is actually well embedded in legal thinking: showing a duty’s impossibility or bizarreness often serves as a political argument against the acceptability of the correlative (human) rights. But if we fail to see clearly who the involved counterparties, the agents are (for example by misunderstanding the enforceability by the State and putting the State in the one of the counterparties’ role), our argument will also fail. For examples see [13]. By my proposal I aim to clarify how and when the State is involved in providing, ensuring and enforcing (human) rights.

2.4. Power’s Formal Representation: Duty-generating Potential with Constitutive Rules behind it

Beside handling counterpartyness Sergot declares formalizing power as the other limitation of the theory of normative positions. As mentioned above, the higher-order property of the second group of rights comes from the fact that "this group is concerned with changes of legal/normative relations"[5]. Fitch’s and Makinson’s findings tell more, though: the formal representation has to give an account the special capacity involved into power and the incapacity to actually do the given act when it lacks. But as Sergot[5]—referring to Makinson[3]—points out in his article on normative positions, “it has long been understood that ‘power’ in the sense of (legal) capacity or ‘competence’ cannot be reduced to permission, and must also be distinguished from the ‘can’ of practical possibility.” Connected to that, Sergot also mentions his paper with Jones[9] in which they “argue that ‘power’ in this Hohfeldian sense is to be understood as a special case of a more general phenomenon, whereby in the context of a given normative system or institution, designated kinds of acts, performed by designated agents in specific circumstances, count as acts that create specific kinds of institutional relations and states of affairs. This switches attention from the formalisation of permission to the formalisation of the counts as relation more generally.” In agreement with this, in my approach I call this capacitative feature of power ‘potential’ and I try to serve this idea of ‘count as’ by capturing this feature as a feature borne by power and acts together. Considerably obvious—as Sartor also discusses them together in his finely tuned right formalization[11] or as those are bond by many others, see in Grossi and Jones[25]—that the nature of rules we have to build the notion ‘count as’ on is the notion of constitutive rules, since these rules constitute an activity the existence of which is logically dependent on the rules [10]. But if we look 40 years earlier, at Rawls we find the exact characterization Makinson points at in the case of power: contrary to summary rules, practice rules are what define an action: if the rules are not followed, we are not engaged in the defined activity [12].

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1This choice might seem at first glance a strict legal positivist’s approach, but actually this restriction is exactly the step through which we avoid such a commitment: we do not describe rights in general in this way, we only define legal rights.
3. Formalization I. Providing Formal Definitions

The first part of my work has been to provide formal definitions for rights and duties aiming to represent formally what Hohfeld meant by differentiating them (see [13] and [14]). In order to do so—I—following Sergot [5]—used SDL as the underlying system with a ‘sees to it that’ operator. Contrary to what most associate with the operator’s name, in Sergot’s (and so far in my) representation of the normative positions’ theory, this operator does not come with a “classic” STIT logic, but the very simple logic Chellas [6] called ET, containing only the rule of equivalents’ changeability and the T-scheme in order to have successful actions. I used the same leaving the emphasis on what the formal definitions aim to tell about the nature of rights and duties. I use this “simplified” STIT operator iterably, since this enables us to capture the real act bound by the deontic operators (denoting the rights and duties). Of course I needed to introduce these to the language in a way they express their being assigned to agents in order to be able to describe counterparties. Using the notion of directedness—introduced by Herrestad and Krogh [15] and others—also seems expressing enough to be worth to involve. This way, the set of agent-indexed modal operators and variables (generals as x and y, and specials as j) representing agents expands in the following way:

\[
\begin{align*}
CR_x & : x \text{ has a claim-right} \\
F_x & : x \text{ has a freedom (privilege)} \\
O_x & : x \text{ has an obligation} \\
O_{x\to y} & : x \text{ has an obligation towards } y \\
P_x & : x \text{ has the power} \\
L_x & : x \text{ has the liability} \\
I_x & : x \text{ has the immunity} \\
D_x & : x \text{ has the disability} \\
E_x & : x \text{ sees to it that} \\
S & : \text{ a given state of affairs} \\
x,y & : \text{ agents} \\
j & : \text{ judge}
\end{align*}
\]

In order to ensure that these operators play the roles we (Hohfeld) intend(s) them to play, we can create syntactic constraints. From a semantic point of view these can be regarded as meaning postulates:

- Having a claim-right is a passive situation, therefore as an argument of the claim-right operator there can be only a STIT operator with a different agent-index. This means that \( CR_x E_x S \) is syntactically wrong, since we can have a claim-right only to someone else’s act.
- Having a freedom is an active situation: it pertains to our own act. Therefore as an argument of the freedom operator there can be only a STIT operator with the same agent-index as the freedom operator itself. This means that \( F_x E_y S \) is syntactically wrong (if \( x \neq y \)).

Instead of giving the definitions in the object language with biconditionals (as I did earlier, following Makinson, see [13] and [14]) it seems more reasonable, clear and uniform to give them as derivation rules. To find it as obvious as it can be, it is useful to
emphasize how we would like to see the logical system describing our world: it contains propositions on plain facts (on simple state of affairs), propositions on acts, and propositions (understand: facts) on ones having rights and duties. In a system like that it is self-explanatory to define a right by telling how we can derive the proposition claiming someone having it, or what we can derive from the fact (proposition) that someone is having it: we define with showing what its logical consequences are.

As exploited above, in the case of the first group of rights and duties we build on enforceability, as Makinson does, but instead of power we use claim-right as a crucial (and thus—in a strict sense—as an undefined) notion. What does it mean to have a claim-right, what kind of success-expectation can we have initiating a litigation? As a biconditional we would say that y has a claim-right to x’s seeing to it that S iff the following holds: if x does not see to it that S, then y has a claim-right to the judge’s seeing to it that x see to it that S (y has a claim-right to the judge’s making x see to it that S). So we add the following derivation rules to our syntax:

\[
\begin{align*}
(1a) & \quad \text{If } \Gamma \vdash CR_yE_xS \text{ then } \Gamma \vdash \neg E_xS \rightarrow CR_yE_j(E_xS) \\
(1b) & \quad \text{If } \Gamma \vdash \neg E_xS \rightarrow CR_yE_j(E_xS), \text{ then } \Gamma \vdash CR_yE_xS
\end{align*}
\]

With deduction rule (which we have in SDL) we get:

\[
\begin{align*}
(1c) & \quad \text{If } \Gamma \vdash CR_yE_xS \text{ and } \Gamma \vdash \neg E_xS, \text{ then } \Gamma \vdash CR_yE_j(E_xS)
\end{align*}
\]

What is \( \Gamma \)? It is the set of premises and axioms. Among this axioms we find logical ones, but also legal ones—we come back later to this point. Given that the correlation between claim-right and duty is always present, the rule can be expanded with the duty—or actually the formula of claim-right can be changed the correlative duty’s formula. In the case of duty the formula has to show that this kind of duty is a directed obligation since it cannot be read otherwise from this part of the formula: we only know that it is about y’s claim-right if the directedness of the duty is indicated in this way:

\[
\begin{align*}
(2c) & \quad \text{If } \Gamma \vdash O_{y \rightarrow y}E_xS \text{ and } \Gamma \vdash \neg E_xS, \text{ then } \Gamma \vdash CR_yE_j(E_xS)
\end{align*}
\]

Within the category of obligation this directedness differentiates duty from no-claim, where the action-logic part of the formula (the indexed STIT operators) designates the other party (the agent of the correlative right).

Describing the nature of freedom can happen in a similar way: I have a freedom to do something iff as soon as someone precludes me from acting according to my freedom, I will have a claim-right towards the judge to stop him from precluding me. But there is a very important difference: in the case of freedom the counterparty is actually everyone else. It is important to stress that we deal with finite agents—but all of them (except x itself), thus we have to give the derivation rule in the following form:

\[
\begin{align*}
(2c) & \quad \text{If } \Gamma \vdash O_{y \rightarrow x}E_xS \text{ and } \Gamma \vdash \neg E_xS, \text{ then } \Gamma \vdash CR_yE_j(E_xS)
\end{align*}
\]

\[\text{2This choice might get a confirmation from legal terminology: in the Hungarian legal language the state in which one's fundamental right gets in case of infringement is called claim ('alapjogi igény') and it is describing one seeking remedy at court: the speciality of legal rights we stressed above as a motivation of the chosen formalization.}\]
Given the correlation between freedom and the no-claim, the formula $\Phi$ can be expressed in the following way too:

$$ (4) \quad \Phi(x, y) : O_y \neg E_y (\neg E_x S) $$

We have not said anything specific yet about the new modal operators’ scope but in the formula above it seems that we want to be consequent in that they effect on actions (that are expressed by STIT operator effecting on state of affairs) so we follow von Wright’s distinction[7] between doing and forbearing/refraining in a way that Belnap–Perloff–Xu did[6] and Sergot[5], and express refraining with iterating the STIT operator.

One could raise the issue that the formulas above are missing the option of appeal, which can have a crucial role in fighting for rights. According to this the formulas can be expanded in the following way (where $J$ stands for the appellate court):

$$ (5) \quad \Gamma \vdash O_x E_x \neg E_x S, \Gamma \vdash \neg E_x S $$

$$ \Gamma \vdash CR_x J(E_x S)(\neg E_x S) \land (\neg E_x S \rightarrow CR_x J(E_x S) \lor E_x S) $$

The disjunction in the consequent of the added conditional shows the two possible types of decision of the appellate court: overwriting the original sentence or remanding the case to the original court for new trial can be the tools for ending up with a decision in accordance with the claim-right. Usually the appeal systems are multiple-stage, but if the nature of the appeal and the possible decisions of the each appellate court do not differ, it does not have to be indicated separately.

The same extension can be done with the notion of freedom.

What about power and the rights and duties connected to it? How are we to express them formally? The formalization has to point out the difference between the two Hohfeldian groups. It is tempting to simply use the consideration made by Makinson[3]: if someone does something without the power (the authority) to do it, he practically does not do it. This “feature” seems to be very important in order to describe what power (at least its lack) is. But the T-scheme of $E_x$ stops us expressing this property in this way:

$$ \neg P_x E_x S \rightarrow (E_x S \land \neg S) $$

Hohfeld provides a lot of examples of power, but the most concrete remark (which can only be found in the version of Hohfeld’s analysis from 1964) we can use for the formalization is about the correlative concept of liability: Hohfeld says that “it is a liability to have a duty created”. With this relation we can formalize liability in the following way:
(6) If $\Gamma \vdash L_yE_xS_c$ and $\Gamma \vdash E_xS_c$, then $\Gamma \vdash O_yE_y\Psi_c(S')$

Which means of course that $x$ has the power to see to it that $S$:

(7) If $\Gamma \vdash P_xE_xS_c$ and $\Gamma \vdash E_xS_c$, then $\Gamma \vdash O_yE_y\Psi_c(S')$

The main question is, obviously: what are $S_c$ and $\Psi_c(S')$? The c in the index is supposed to indicate that we can have power to see to it states of affairs which are constituted by the law: by constitutive rules. $\Psi$ stands for a function which shows how constitutive rules define relations between specific states of affairs (or seeing to that these are the cases, i.e., specific acts). But what kind of act and states are these? In order to answer this question, first we need to call up some examples: acts which we can say one has the power (authority) to perform. Hohfeld mentions transferring one’s interest, extinguishing of legal interest, agency relations and options, we can add for example eviction, search (of premises), garnishment or other types of execution. Should we use these as values for $S$? Sounds reasonable since—knowing the Hohfeldian system—saying things like ‘one has the power to execute’, ‘one has the power to transfer his interest’ or ‘one has the power to evict someone else’ makes sense. But then what is $S'$? If $S$ is eviction then $E_xS'$ should stand for $y$ leaves her house. But it sounds a little bit strange that if $x$ sees to it that the eviction is done (if $x$ evicts) then $y$ has the obligation to leave her house. The source of the strangeness is that if $x$ has already done the eviction then $y$ has already left her house, so there is no obligation of her to leave it. Of course we can say that she has the duty to keep away, but when we talk about eviction, we primarily think her duty to leave as the one which comes due to the eviction. That is how we imagine the eviction itself: to order her to leave her house. Accordingly, my suggestion is to consider $E_xS$ as something simpler, more elementary instead of executing the eviction. It should rather be saying something—in case of eviction saying that $y$ has to leave the house. That is, $S$ is an utterance, it is that something is said (or written). The something that is said is not independent from $S$. My proposal formally is the following:

(8) $\Psi_c : S := U(O_yE_yS' \land O_xE_xS'' \land O_wE_wS''' \land ...)$

that is, $S$ is an utterance of $y$’s obligation to see to it that $S'$ and $v$’s obligation to see to it that $S''$ and $w$’s obligation to see to it that $S'''$ and so on (the notation ‘...’ does not mean that the formula is infinite, only that we cannot tell how long it is.) One could raise here that maybe in the case of eviction the sentence ‘$y$ has to leave the house’ is really said, but if $x$ wants to sell his car, he will say only ‘I sell it’. But actually saying ‘I sell it’ is just an abbreviation: it contains a set of changes in rights—which changes are listed in the sales contract. Just as the duty of $y$ to leave is written in the eviction order. These abbreviations are the names or labels of the legal or institutional constructions (which can be different in the different legal systems). $E_xS_c$ is the conception of making a legal statement. Therefore—and it is important to lay down that—these relations, these abbreviations of the given legal system are given by legal, professional axioms, not by logical ones, so it is not a task of the logical calculus to say more about them (which means, unlike it is usual as it can be seen in Grossi and Jones[25], I do not see necessary to use a separate logic to them). These legal axioms will serve as premises in the set of premises
for which we referred earlier as $\Gamma$.

The T-scheme for $E_x$ does not render this formalization vacuous since it cannot be the case that someone sees to it that $S$ and not $S$, but it can be the case that someone utter $S$ but it won’t be the case that $S$. This is what we use: to be able to have the consequent (of saying $S$) that $y$ has a duty, $x$ needs to have the power. Standing next to the Trump Tower I can utter the sentence that ‘I transfer it to an Ethiopian orphan’, but—unfortunately—nothing will happen. The reason for this lack of consequence is that I don’t have the power, that is, I have a disability considering the selling of the tower.

With this approach we don’t need to deny the derivability of $y$’s duty in case of $x$’s disability: actually we don’t want to deny it, since her duty can come from somewhere else. It is enough to say that:

$$(13) \quad I_y E_x S \overset{\phi \in L_y E_x S} {\rightarrow} \neg I_y E_x S_c$$

4. The role of the State

In these formulas there is no agent called ‘State’. What is the role of the state then? One—obvious—option is to consider the judge as the agent who acts in the name of the State. But this only means the judiciary—which is only one power (now in a Montesquieunian sense) of the State. Not only that one is involved, though: it is the legislation which has the task (and responsibility in case of undertakings in international contracts) in ensuring rights. When a constitution says that the State guarantees a right, what does it mean? It means that the State guarantees the derivation rules above. How can a State guarantee derivation rules? By creating a legal system that obeys a logic in whose calculus contains these derivation rules: having a legal system which can be described by a logic in which has the derivation rules given above.

Could it be otherwise? Well, the usual comment here is that legal validity is actually insensitive to logical validity. A legal system can be legally valid without maintaining consistency among rights and duties. But does that legal system really contain those rights? My answer is ‘no’. To say a legal system contains a right means that the legal system obeys a logic which contains the given derivation rules. And this is what is expressed in the Declaration of the Rights of Man and Citizen’ cited paragraph.

5. Formalization II. Providing Semantics and Axiomatization

Meanwhile it seems to be practical to stick to an already existing system (in our case SDL) and just add some formal definitions in order to avoid any complications or the bur-

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3It’s better to say ‘legal system’ instead of ‘system of norms’ since by and in the formulas we considered legislation and judicature too.

4Here we consider legal validity practically as the existence of a legal norm: it has been created in the prescribed way (procedure), by the agency who has the power to create it. If the prescribed procedure doesn’t say anything specific about logical requirements then legal norms can exist (which means that they are valid—as von Wright assumes too from the very beginning) without being consistent.
den of building a new system, there are several arguments pursuing us acting differently: (i) SDL is a pretty problematic system, as Sergot puts it: "it has many well-known limitations and its inadequacies are taken as the starting point for many of the developments in the field", and the top of it all, in our new derivation rules we don’t really build on its specialties; (ii) it also can be raised that the action operator used above is too simplified compared to the last decades’ developments of action logic, so it is worthy to make some considerations on using a "real" STIT operator with its logic behind (especially if we insist to the thought that there must be some underlying temporal logic behind); (iii) without semantics we do not model the Hohfeldian system. In the light of these considerations, it seems reasonable to proceed with working out a system that leaves SDL behind. Of course, a pursuit like this involves a lot of questions. It seems reasonable to choose the action operator and build the semantics first using the operator’s semantics, and just then go back to syntax and try to provide an axiomatization. Choosing the right action operator is crucial since it appoints the frame we have to work in. In this decision it seems plausible to lean on such works’ choices in which action theory was involved in some kind of deontic logic, like H Horty[16], Pörn[17] and others, also on classics of action logic like Chellas[4] and Belnap and Perloff[18], on systematizing works like Hilpinen [19] or Segerberg[20], or on the new developments in it like Troquard[21]. And since handling different (presumably normal and non-normal) modal semantics in an integrated system is a challenge, relying on pioneer works like Calardo and Rotolo’s work on multi-relational semantics.[22] Once the result would be a complex system with semantics and syntax adequate for AI application, it could be compared with such AI related languages as the LLD[23] and the A-Hohfeld[24].

References

[21] N. Troquard: Reasoning about coalitional agency and ability in the logics of "bringing-it-about". In: Autonomous Multi-Agent Systems, 28/3