Full Name:____
(block capitals)

Student Number:______________Tutorial Group Number:____

4CCM122A Geometry I: Test 3

CALCULATORS MAY NOT BE USED

ANSWER GRID: put a cross in ONE BOX for the correct answer for each question. If you change your mind and want to correct your answer, obliterate your incorrect answer by shading its box, and put a new cross in the box for the correct answer.

	a	b	c	d	e
1					
2					
3					
4					

MARKS: each correct answer = +5, incorrect = −1, none (or more than one) = 0.

Do any rough working on the back of this sheet, or on a NAMED separate sheet. You are strongly advised to draw diagrams.

1. Let f be the isometry defined by the rule $f(z) = iz + 6 + 6i$.
Which one of the following points is a fixed point of f?

(a) $6i$ (b) -6 (c) $3+3i$ (d) $3-3i$
(e) 6+6i

2. The line $x + y - 1 = 0$ is the mirror line of which one of the following reflections?

(a) $f(x, y) = (y + 1, x - 1)$
(b) $f(x, y) = (y, x)$
(c) $f(x, y) = (-y, -x)$
(d) $f(x, y) = (1 - y, -x + 1)$
(e) $f(x, y) = (2 - y, 2 - x)$

3. Let g be the counter-clockwise rotation defined by $g(z) = \frac{\sqrt{2}}{2}(1 + i)z$. Then the angle of the rotation is

(a) π (b) $-\frac{\pi}{2}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{4}$
(e) $-\frac{3\pi}{4}$

4. One and only one of the maps below is an isometry. Which one?

(a) z^2
(b) $|z|$
(c) $5z$
(d) $i\bar{z} + 2 - i$
(e) $\sin|z|$

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Solutions

	a	b	c	d	e
1	×				
2				×	
3				×	
4				×	

Note: Your answers, if correct, will *not* have given the above pattern, because (as a guard against cheating) there were several versions of the question paper, with the possible answers arranged in various orders.

1. (a) since $f(6i) = -6 + 6 + 6i = 6i$ while $f(-6) = -6i + 6 + 6i = 6$, $f(3 + 3i) = 3i - 3 + 6 + 6i = 3 + 9i$, $f(3 - 3i) = 3i + 3 + 6 + 6i = 9 + 9i$ and $f(6 + 6i) = 6i - 6 + 6 + 6i = 12i$.

2. (d)One solution: $(1, 0)$ and $(0, 1)$ lie on the mirror line. An isometry f is reflection in $x + y - 1 = 0$ if and only if $f(1, 0) = (1, 0)$ and $f(0, 1) = (0, 1)$.

Now if (a) $f(x, y) = (y + 1, x - 1)$ then $f(0, 1) = (2, -1)$, if (b) $f(x, y) = (y, x)$ then $f(1, 0) = (0, 1)$, if (c) $f(x, y) = (-y, -x)$ then $f(1, 0) = (0, -1)$ and if (e) $f(x, y) = (2 - y, 2 - x)$ then $f(1, 0) = (2, 1)$

but if $f(x, y) = (1 - y, -x + 1)$ then $f(1, 0) = (1, 0)$ and $f(0, 1) = (0, 1)$.

A better method: The point (x, y) is a fixed point of the reflection in (a) if and only if $x = y + 1$ and $y = x - 1$. Thus the line $x - y - 1 = 0$ is a line of fixed points for the reflection, and so must be the mirror line. Similarly $x - y = 0$ is the mirror line for (b), $x + y = 0$ is the mirror line for (c), $x + y - 1 = 0$ (the required line) is the mirror line for (d) and $x + y - 2 = 0$ is the mirror line for (e).

3. (d) First, let's show that g is a rotation. The only solution of $g(z) = z$ is $z = 0$. Therefore, 0 is the only fixed point and g is a rotation centered at the origin. To find the angle, compute $g(1) = \frac{\sqrt{2}}{2}(1 + i)$, therefore the angle of rotation is $\frac{\pi}{4}$.

4. (d) By ruling out the other options. Not (a) because $f(0) = 0$ and $f(2) = 4$. Not (b) because $f(i) = f(1) = 1$. Not (c) because $f(0) = 0$ and $f(1) = 5$. Not (e) because $\sin(0) = \sin(\pi) = 0$. Remember, an isometry is injective.

Tinaglia—1 December 2008