.

FULL NAME:_____
(BLOCK CAPITALS)

STUDENT NUMBER:_____ TUTORIAL GROUP NUMBER:____

4CCM122A Geometry I: Test 3

CALCULATORS MAY NOT BE USED

ANSWER GRID: put a cross in ONE BOX for the correct answer for each question. If you change your mind and want to correct your answer, obliterate your incorrect answer by shading its box, and put a new cross in the box for the correct answer.

	a	b	c	d	e
1					
2					
3					
4					

MARKS: each correct answer = +5, incorrect = −1, none (or more than one) = 0.

Do any rough working on the back of this sheet, or on a NAMED separate sheet. You are strongly advised to draw diagrams.

1. Let T_1 and T_2 be two isometries and let P be a point such that $T_1(P) = T_2(P)$. Suppose that for any $Q \neq P$, $T_1(Q) \neq T_2(Q)$. Then there exists an isometry T_3 such that $T_1 = T_3 \circ T_2$ and ... (complete the sentence)

(a) T_3 is a rotation.
(b) T_3 is a translation.
(c) T_3 is a reflection across a line.
(d) T_3 is a reflection after a translation
(e) None of the above.

2. The line $x - y + 1 = 0$ is the line of reflection of which one of the following reflections?

(a) $f(x, y) = (y - 1, x - 1)$
(b) $f(x, y) = (y, x)$
(c) $f(x, y) = (-y, -x)$
(d) $f(x, y) = (y - 1, x + 1)$
(e) $f(x, y) = (2 - y, 2 - x)$

3. Let g be the counter-clockwise rotation defined by $g(z) = iz + 1 - i$. Then the angle of the rotation is

(a) π (b) $-\frac{\pi}{4}$ (c) $\frac{\pi}{3}$ (d) $\frac{\pi}{2}$
(e) $-\frac{3\pi}{4}$

4. One and only one of the maps below is an isometry. Which one?

(a) $\cos|z|$
(b) $\frac{|z|}{2}$
(c) z^3
(d) $\frac{\sqrt{2}}{2}(1 + i)\bar{z} + 2 - i$
(e) $\frac{z}{3}$

END OF TEST 2010/11

Solutions

	a	b	c	d	e
1	×				
2				×	
3				×	
4				×	

Note: Your answers, if correct, will *not* have given the above pattern, because (as a guard against cheating) there were several versions of the question paper, with the possible answers arranged in various orders.

1. (a) $T_3 = T_2^{-1} \circ T_1$. The hypotheses give that P is the only fixed point of T_3 therefore T_3 is a rotation.

2. (d)$(-1, 0)$ and $(0, 1)$ lie on the line. An isometry f, which is not the identity, is a reflection across $x - y + 1 = 0$ if and only if $f(-1, 0) = (-1, 0)$ and $f(0, 1) = (0, 1)$.

3. (d) The center of the rotation is the solution of $z = g(z)$, that is 1. $g(0) = 1-i$ therefore the answer is (d).

4. (d) By ruling out the other options. Recall that an isometry is a bijection. $\cos|z|$ and $\frac{|z|}{2}$ are not bijections. z^3 sends 0 into 0 and 2 into 8, and $\frac{z}{3}$ sends 0 into 0 and 3 into 1.

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