Geometry I
(CM122A, 5CCM122B, 4CCM122A)

- Lecturer: **Giuseppe Tinaglia**
- Office: **S5.31**
- Office Hours: **Wed 1-3** or by **appointment**.
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- Course webpage: http://www.mth.kcl.ac.uk/~tinaglia/geometry

In this class

- Euclidean Geometry, Birkhoff's axioms.
- Hyperbolic Geometry.
- Develop the ability to construct mathematical arguments.

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Books

- Euclidean Geometry
  - G. D. Birkhoff and R. Beatley: **Basic Geometry**.
  - J. R. Silvester: **Geometry Ancient and Modern**.
- Hyperbolic Geometry
  - D. A. Brannan, M. F. Esplen and J. J. Gray: **Geometry**.
- Very interesting reading
  - M. J. Greenberg: **Euclidean and Non-Euclidean Geometries, Development and History**.

Homework

- **Homework** will be posted every **Monday** on the class webpage.
- Turn it in **next Thursday** to your tutor.
- You will be able to ask questions on it the **following Thursday** during tutorial.
- Walk-in tutorials on **Wednesday** at 2.00pm in S5.21 starting Wed 13th October.

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Examination

- There are **4 multiple-choice tests** held during the semester which count for 20% of the final marks.
- The remaining 80% of the marks are assessed by a **three-hour written examination** in May.

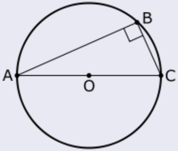
Let none ignorant of geometry enter here.
Written above the entrance to Plato's school, 427-347 BC

Geometry:

- Literally **Earth-measuring**.
- Collection of empirically discovered **principles** concerning lengths, angles, areas, and volumes.
- **3000 BC**, India and Mesopotamia.

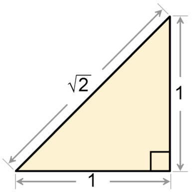
Thales of Miletus, 624-c. 546 BC

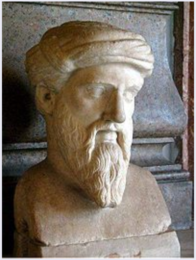
- Replaced experience with **deductive reasoning**.
- First true mathematician.
- Thales Theorem,



Pythagoras of Samos, c. 570-c. 495 BC

- Very famous Pythagorean school.
- $\sqrt{2}$ is not rational.
- Let's do **geometry**,





Euclid of Alexandria, fl. 300 BC

- Organized much of what was known in geometry at the time in 13 books, **the Elements**.

The Elements

- Masterpiece of logic — **Axiomatic Method**.
- Most durable and influential book in mathematics.
- Second only to the Bible in the number of editions published.

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Assuming a small set of intuitively appealing **axioms** (**rules of the game**), deduce many other propositions.
Point, straight line and circle are taken as undefined terms.

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- It is possible to draw a straight line from any point to any other point.
- It is possible to extend a line segment continuously in a straight line.
- It is possible to describe a circle with any center and any radius.
- It is true that all right angles are equal to one another.

These axioms come somewhat from **experience**.

Euclid’s Fifth Postulates—Parallel Postulate

- Given a line **l** and a point **P** not on it, there is one and only one line **m** through **P** which does not meet **l**. (**Playfair’s axiom**)
- The line **m** is the **unique** line **parallel** to **l**.
- **Existence** and **Uniqueness**.

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This set of axioms is **incomplete**. Let us construct an isosceles triangle.

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David Hilbert (January 23, 1862-February 14, 1943)

- The Foundations of Geometry
- 20 axioms

Parallel Postulate

Through a given point not on a given line there is one and only one line which does not meet the given line.

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Questions

- Is the **Parallel Postulate** really a **necessary** rule of the game?
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Questions

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- What does it mean to be a "**necessary**" rule?

Answer

- If we can **deduce** the Parallel Postulate from the other axioms then such rule is **NOT necessary**.
- Replace the Parallel Postulate with
  - **there exists no parallel line** or
  - **there exist more than one parallel line**

  and obtain a contradiction.

On the other hand, if we can change the Parallel Postulate into:

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and we never, **EVER**, incur into a contradiction then we have shown the existence of **other geometries!**

Are there other geometries?

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Yes, and they are called **Non-Euclidean Geometries**. Such discovery was (and still is) a big deal.

Founding Fathers of Non-Euclidean Geometries

Gauss
1777-1855

Lobachevsky
1792-1856

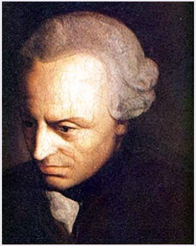
Bolyai
1802-1860

- No parallel line exists, **elliptic geometry.**
- More than one parallel line exist, **hyperbolic geometry.**

Why was the discovery of **Non-Euclidean Geometries** a big deal?

Immanuel Kant (April 22, 1724-12 February 12, 1804)

- He said that space is **objective**, a priori, and it is Euclidean.
- The existence of **Non-Euclidean Geometry** strikes the **first blow** to this philosophy.
- **General Relativity** puts the final nail in the coffin.



Wait a minute...

- Do **Non-Euclidean Geometries** really exist? Really?

- Can we change the Parallel Postulate into
  - there exists no parallel line or
  - there exist more than one parallel line

  prove theorems and do NOT incur into a contradiction,

  EVER?

Wait a minute...

- Are Non-Euclidean Geometries **consistent**?

- **Non-Euclidean Geometries** are consistent if and only if **Euclidean Geometry** is consistent if and only if **Arithmetic** is consistent.

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Wait a minute...

- Is **Euclidean Geometry** consistent?

- We do **NOT** know.

- We **CANNOT** know!

Kurt Godel (April 28, 1906-January 14, 1978)

- No "sufficiently complex" system which is consistent can prove its consistency.
- Sufficiently complex: being able to define arithmetic is more than enough.

Bertrand Russell (May 18, 1872-February 2, 1970)

- **Mathematics** is the only science where one never knows what one is talking about nor whether what is said is true.