Geometry I — Homework 7 — Due 2nd Dec

1. Let ABC be a triangle inscribed in a circle and let P be a point in $arcBC$. Show that $\angle A = \angle PBC + \angle PCB$.

2. Let ABC be a triangle and let M in AB and N in AC such that CM bisects $\angle C$ and BN bisects $\angle B$. Let $D \in BN$ and $E \in CM$ such that $AD \perp BN$ and $AE \perp CM$. Prove that $DE \parallel CB$. (Hint: extend AD and AE to intersect the line containing BC in D' and E'. Show that AED and $AE'D'$ are similar.)

3. Let l and m be two distinct lines intersecting at a point P. Prove that if a circle Γ is tangent to both lines then its center lies on one of the bisectors of the angles formed by such lines.

4. Show that given a circle Γ and a point A outside Γ there exist exactly two lines containing A and tangent to Γ. (Hint: Let l be a tangent line, let O be the center of the circle, let $P \in \Gamma \cap l$ and let $B \in AO \cap l$. Let m be the tangent line to Γ at B. Studying $m \cap l$ prove first that there can only be two such lines (uniqueness). Then, find a way to construct them (existence).)