Geometry I — Homework 5 — Due 18th Nov

1. Let $ABCD$ be a quadrilateral. Prove that $ABCD$ is a parallelogram (opposite sides are parallel) if and only if opposite angles are equal.

2. Prove that the diagonals of a parallelogram $ABCD$ bisect each other.

3. Prove that if the diagonals of a quadrilateral $ABCD$ bisect each other then it is a parallelogram.

4. A quadrilateral having two and only two sides parallel is a trapezoid. Let $ABCD$ be a trapezoid, $AD \parallel BC$ and let E and F be respectively the midpoint of AB and CD. Prove that EF is parallel to BC. (Hint: let H be the midpoint of AC. Show that EH and HF are parallel to BC. Conclude that E, H and F are collinear (on the same line) and that EF is parallel to BC.)

5. Let ABC be a triangle and let BS be the bisector of $\angle B$ with S in AC. Prove that $AS/SC = AB/BC$. (Hint: extend BC in the direction of B and let R be the intersection between such extension and the parallel to BS through A.)