Geometry I — Homework 4 — Due 4th Nov

1. Let ABC be a right triangle such that $\angle A = 90°$. Show that if $BC = 2AB$ then $\angle B = 60°$.

   Sol: Extend AB of a segment AD such that $AD = AB$. By (SAS) ABC and ADC are similar. Therefore, $BC = BD = DC$ and BCD is an equilateral triangle.

2. Let ABC be a triangle such that $AB = AC = 2$ and $BC = \sqrt{8}$. What are $\angle A$, $\angle B$ and $\angle C$?

   Sol: By the Pythagorean Theorem, ABC is a right triangle, $\angle A = 90°$. Since $AB = AC$, it is also an isosceles triangle. Thus $\angle B = \angle C = 45°$.

3. Prove that the sum of the angles of a quadrilateral polygon is 360°.

   Sol: Divide the polygon into two triangles.

4. Let ABC be a right triangle, $\angle C = 90°$, and let CD be the altitude at the vertex C. Show that $CD^2 = AD \times DB$.

   Sol: Applying the Pythagorean Theorem, $AB^2 = AC^2 + BC^2$, $CB^2 = CD^2 + DB^2$ and $AC^2 = CD^2 + AD^2$. Since, $AB^2 = (AD + DB)^2 = AD^2 + DB^2 + 2AD \times DB$, then $AD^2 + DB^2 + 2AD \times DB = CD^2 + DB^2 + CD^2 + AD^2$.

5. Show that a triangle with sides $p^2 - q^2$, $2pq$ and $p^2 + q^2$, where $p > q$ is a right triangle.

   Sol: $(p^2 + q^2)^2 = p^4 + 2p^2q^2 + q^4 = (p^2 - q^2)^2 + (2pq)^2$. Therefore it follows from Pythagorean Theorem.

6. Let ABC and $A'B'C'$ be triangle such that $AB = A'B'$, $BC = B'C'$ and $\angle A = \angle A'$. Show that they are equal.

   (Hint: Let BH and $B'H'$ be the altitudes from B and B'...)

   Sol: $ABH = A'B'H'$ by (AAA). Therefore $BH = B'H'$ and, using the Pythagorean Theorem, $HC = H'C'$. Thus $BHC = B'H'C'$ by (SSS). Therefore $ABC = A'B'C'$ by (SSS).

7. Let $ABCD$ be a quadrilateral such that $AB = AD$ and $CB = CD$. Show that AC is a segment on the perpendicular bisector of BD.

   Sol: The points A and D are equidistant from the end-points of BC therefore they are contained in the perpendicular bisector(proved in class). Since through 2 distinct points there exists a unique line...