Geometry I — Homework 2 — Due 21st Oct

1. Convince yourselves that the Cartesian plane satisfies the postulates of Euclidean geometry.

2. Write down the negation of each of the following statements:

   - All numbers are prime numbers.
   - Some triangles are isosceles triangle.
   - There are no right angles.

   Sol:

   - Some numbers are not prime.
   - All triangles are not isosceles triangle.
   - Some angles are right angles.

3. Let AB and CD be two line segments intersecting at V. Show that the lines bisecting $\angle AVC$ and $\angle CVB$ are perpendicular.

   Sol: Let VP and VQ be the half-lines bisecting respectively $\angle AVC$ and $\angle CVB$, then $\angle PVC = 1/2\angle AVC$ and $\angle CVQ = 1/2\angle CVB$. Thus $\angle PVQ = \angle PVC + \angle CVQ = 1/2(\angle AVC + \angle CVB) = 1/2(\angle AVB) = 1/2(180°)$.

4. Let ABC and $A'B'C'$ be two triangles such that $AB = 6, BC = 8, \angle ABC = 62°$ and $A'B' = 9, B'C' = 12, \angle A'B'C' = 62°$. What is $C'A'/CA$ and why?

   Sol: Using the SAS criteria for similarity (postulate 4) gives that ABC and $A'B'C'$ are similar. Thus $C'A'/CA = B'A'/BA = 3/2$

5. Let ABC and $A'B'C'$ be two triangles such that $AB = 5, BC = 7, \angle ABC = \alpha°, \angle BCA = \beta°$ and $A'B' = 10, B'C' = 14, \angle A'B'C' = \alpha°$. What is $\angle B'C'A'$ and why?

   Sol: Like before the triangles are similar and thus $\angle B'C'A' = \angle BCA = \beta$.

6. Definition: A polygon with four sides is called a *quadrilateral*. A quadrilateral is *convex* it all its angles measure less than 180°.

   Let $ABCD$ and $A'B'C'D'$ be two similar, convex quadrilaterals such that $\angle ABC = \angle A'B'C', \angle BCD = \angle B'C'D', \angle CDA = \angle C'D'A', \angle DAB = \angle D'A'B'$. Prove that the triangles ABC and $A'B'C'$ are similar.

   Sol: Since $ABCD$ and $A'B'C'D'$ are similar, $AB/A'B' = BC/B'C'$ and $\angle ABC = \angle A'B'C'$. Thus. SAS gives that ABC and $A'B'C'$ are similar.

7. Consider a triangle ABC and let L, M, N be respectively the midpoints of AB, AC and BC. Show that the triangles ALM, LBN, CMN and ABC are similar.

   Sol: Let us show that ALM and ABC are similar. $AB/AL = AC/AM = 2$ and $\angle ALM = \angle ABC$. Thus, they are similar because of SAS.