Geometry I — Homework 1 — Due 14th Oct

1. Write down the negation of each of the following statements:

   - All people living in Neasden have black hair.
   - Some TV programmes are good.
   - Jeremy Clarkson and Amy Winehouse are shy.
   - Donald Duck always wears glasses and a hat.
   - There is a tree in England whose number of leaves is not equal to the number of words in any book.
   - There is a lecture room such that, for any time of day, there are students able to attend lectures at that time who cannot fit into the room.
   - All triangle have three sides.
   - Given a line l, a point P in l and a number $n > 0$ there exactly two points in l at distance n from P.

   Sol:

   - Somebody in Neasden does not have black hair.
   - All TV programmes are bad.
   - Either JC or AW or both are not shy.
   - DD sometimes does not wear glasses, or does not wear a hat or does not wear either.
   - For any tree in England there is a book whose number of words is equal to the number of leaves on that tree.
   - For any lecture room there is a time of day such that all students able to attend lectures at that time can fit into the room.
   - Some triangle does not have three sided.
   - There exist a line l, a point P in l and a number $n > 0$ such that the number of points in l at distance n from P is not two (could be less or greater).

2. Explain why each of the second statements does not follow from the premise.

   - All roads going south lead to Rome. This road goes west and therefore does not lead to Rome.
   - Most Italians like football. Mario is Italian therefore he likes football.

Sol:

- The premise does not say anything about roads going west.
- The premise does not say all Italians like football.

3. Use Postulate 1 to prove the following: Given a line l, a point P in l and a number $n > 0$ there exist exactly two points in l at distance n from P.

Sol: Let n_P be the real number associated with P via (P1). Then, using the properties of real numbers there exist exactly 2 different numbers n_1, n_2 such that $|n_p - n_1| = |n_p - n_2| = n$. Thus, via (P1), there exist exactly two distinct points P_1, P_2 in l corresponding respectively to n_1 and n_2 such that the distance from P to P_1 (or P_2) is equal to $n_p - n_1$ (or $n_p - n_2$).