Geometry I — Homework 10 — Not Due

Recall:

- The general form of a Euclidean isometry is $az + b$ or $a\bar{z} + b$ where both a and b are complex numbers and $|a| = 1$.
- The equation of a hyperbolic line of the Poincaré disk which is not a diameter is $x^2 + y^2 - 2ax - 2by + 1 = 0$, where a and b are real number such that $a^2 + b^2 > 1$.

1. Find the isometries f that send the points 0, 1 and i, into
   (a) f(0)=1, f(1)=2, f(i)=i+1
   (b) f(0)=2i, f(1)=1+2i, f(i)=i
   (c) f(0)=0, f(1)=i, f(i)=-1
2. Find the equations of the hyperbolic lines of the Poincaré disk containing the points:
   (a) $(\frac{1}{4}, 0), (\frac{1}{2}, \frac{1}{2})$
   (b) $(\frac{1}{4}, \frac{3}{4}), (\frac{1}{5}, \frac{3}{5})$
   (c) $(\frac{1}{5}, \frac{1}{2}), (0, \frac{1}{3})$
3. Prove that the isometry $f(z) = \bar{z} + 1$ is neither a rotation, nor a reflection across a line, nor a translation.
4. Prove that the isometry $f(z) = -\bar{z} + 2$ is a reflection across a line.