To solve a CFT:

Need set of fields φ_i $\{h_i, \bar{h}_i\}$ dimensions

3 pt couplings C_{ijk}

Then everything is determined.

How?

Example: $c = 1/2$ allowed h-values: $\{0, \frac{1}{16}, \frac{1}{2}\}$

Various field combinations are allowed:

$\{(0,0),\ (\frac{1}{2},0),\ (0,\frac{1}{2}),\ (\frac{1}{2},\frac{1}{2})\}$ "Free fermion"

$\quad \mathbb{1} \quad \psi \quad \bar{\psi} \quad :\psi\bar{\psi}:$

$\{(0,0),\ (\frac{1}{16},\frac{1}{16}),\ (\frac{1}{2},\frac{1}{2})\}$ "Ising model"

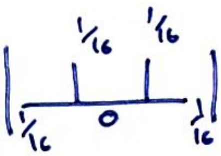
$\quad \mathbb{1} \quad \sigma \quad \varepsilon$

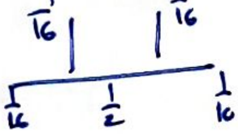
Ising: Can show (using null vectors!) only allowed

3 pt couplings are $C_{\sigma\sigma\mathbb{1}}$ $C_{\sigma\sigma\varepsilon}$ $C_{\varepsilon\varepsilon\mathbb{1}}$

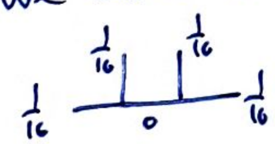
(2 pt couplings) $= 1$.

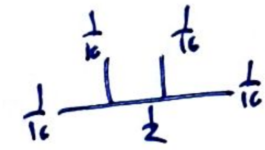
Then

$$\langle\sigma|\, \sigma(1)\, \sigma(z,\bar{z})\, |\sigma\rangle = (C_{\sigma\sigma 1})^2 \left| \quad \right|^2$$ 

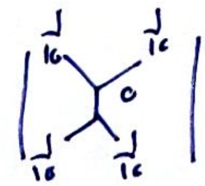
$$+ (C_{\sigma\sigma\varepsilon})^2 \left| \quad \right|^2$$ 

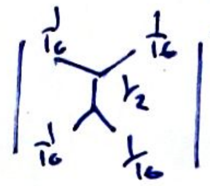
We know these from before.

 $$= \frac{1}{(z(1-z))^{\frac{1}{8}}} \sqrt{\frac{1+\sqrt{1-z}}{2}}$$

 $$= \frac{1}{(z(1-z))^{\frac{1}{8}}} \sqrt{2(1-\sqrt{1-z})}$$

Constraint:

$$\langle \sigma | \sigma(1,1)\, \sigma(z,\bar z)\, | \sigma \rangle = (C_{\sigma\sigma 1})^2 \Big|$$  $$\Big|^2$$

$$+ \; (C_{\sigma\sigma\varepsilon})^2 \Big|$$  $$\Big|^2$$

<u>Need</u> :

$$\frac{1}{|z(1-z)|^{\frac14}} \left\{ \left| \frac{1+\sqrt{1-z}}{2} \right| + C_{\sigma\sigma\varepsilon}^2 \left| 2\left(1-\sqrt{1-z}\right) \right| \right\}$$

$$= \frac{1}{|z(1-z)|^{1/4}} \left\{ \left| \frac{1+\sqrt{z}}{2} \right| + C_{\sigma\sigma\varepsilon}^2 \left| 2\left(1-\sqrt{z}\right) \right| \right\}$$

Fact:
$$\sqrt{1+\sqrt{1-z}} = \frac{1}{\sqrt2}\left(\sqrt{1+\sqrt z} + \sqrt{1-\sqrt z}\right)$$
$$\sqrt{1-\sqrt{1-z}} = \frac{1}{\sqrt2}\left(\sqrt{1+\sqrt z} - \sqrt{1-\sqrt z}\right)$$
$\sqrt{\ }$ missed earlier

$$\Rightarrow \quad C_{\sigma\sigma\varepsilon}^2 = \frac14, \qquad \text{choose } C_{\sigma\sigma\varepsilon} = \frac12 \quad (\text{sign of } \varepsilon).$$

Theory is now fully determined.

- Did not need a Lagrangian, an action, just symmetry, unitarity, elementary properties of QFT
  "Conformal Bootstrap".