"New" Conformal Bootstrap.

Use purely global conformal invariance,

global conformal blocks $\left(z^{h-h_3-h_4} F(h-h_{12}, h+h_{34}, 2h, z) \right)$

Idea: Make an assumption $\Rightarrow$ contradiction $\Rightarrow$ result.

Idea: Make the best (wrong) assumptions $\Rightarrow$ best result.

Simple Example:

Assume there is a scalar field ϕ scale dim d

It couples to the identity $\mathbb{1}$.

Every other field it couples to has scale dim

$$\Delta \gg d$$

Behaviour of blocks will lead to contradiction.

Use crossing symmetry.

$$\langle \phi | \phi(1) \phi(z) | \phi \rangle = \frac{1}{z^{2d}} + \sum_o \lambda_o^2 \left(\frac{11}{\Delta} \right)^{(z,\bar{z})}_{\text{global}}$$

Take z real

$$= \frac{1}{|1-z|^{2d}} + \sum_o \lambda_o^2 \left(\frac{11}{\Delta} \right)^{(1-z, 1-\bar{z})}_{\text{global}}$$

Expand around $z = 1/2$.

$$(z)^{2d} = \left(\tfrac{1}{2}+\varepsilon\right)^{2d} = 2^{-2d}(1 + 4d\varepsilon + \dots)$$

$$(z)^{2d} - (1-z)^{2d} = \underbrace{(8d\,2^{-2d})}_{C_d}\left(\varepsilon + \tfrac{4}{3}(d-1)(2d-1)\varepsilon^3 + \dots\right)$$

$$= \sum_{O} \lambda_O^2 \left[(z)^{2d}\left(\frac{11}{\Delta}\right)^{(1-z)}_{global} - (1-z)^{2d}\left(\frac{11}{\Delta}\right)^{z}_{global} \right]$$

$\underline{\text{If } \Delta \gg d}$ $\quad \simeq \sum_{O} \lambda_O^2 \cdot A_O \cdot \left(\varepsilon + \tfrac{4}{3}\Delta^2\varepsilon^3 + \dots\right)$

$$\Rightarrow \quad C_d = \sum \lambda_O^2 A_O$$

$$C_d(d-1)(2d-1) = \sum \lambda_O^2 \Delta^2 A_O$$

$$\Delta^2_{min} = \frac{\sum \lambda_O^2 A_O \Delta^2_{min}}{\sum \lambda_O^2 A_O} < \frac{\sum \lambda_O^2 A_O \Delta^2}{\sum \lambda_O^2 A_O} = \frac{C_d(d-1)(2d-1)}{C_d}$$

$\Delta_{min} < \sqrt{(d-1)(2d-1)}$ $\quad$ But $\Delta_{min} \gg d$.

Rephrase as $\quad \sum \lambda_O^2 \underbrace{\left[A_O(\Delta^2 - \Delta^2_{min})\right]}_{+ve} + \underbrace{C_d}_{+ve}\underbrace{\left(\Delta^2_{min} - (d-1)(d-2)\right)}_{+ve} = 0$

No soln for $\lambda_O^2 \geq 0$.

Idea: Find sets of equations which have no soln.

Not mentioned

Relation to AdS/CFT

Eigenstate thermalisation hypothesis

Integral representations

Light-heavy blocks

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