NON INVERTIBLE SYMMETRIES FROM DISCRETE GAUGING AND COMPLETENESS OF THE SPECTRUM

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MEETING ON DEFECTS AND SYMMETRY – KING'S COLLEGE LONDON

24^{th} June 2022

Based on 2204.07523 with Diego Rodríguez Gómez

GENERALIZED SYMMETRIES

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- Allows several generalizations:
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  - Higher-group symmetries
  - Non-invertible symmetries ←
- Fusion of topological operators ~ Operator Product Expansion

$$U_a(M) \cdot U_b(M) = \sum \mathrm{coef}_{ab}^{c} \, U_c(M)$$

PRINCIPAL EXTENSIONS

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- Two questions:

  Q_{old} : String theory construction?

  Q_{new} : Higher-form symmetries?

OUTLINE

1. Introduction

2. The (electric) 1-form symmetry

3. The (dual) (d-2)-form symmetry

4. Brane constructions and Alice strings

5. Conclusion

1-FORM SYMMETRY

- Operators:
  - Topological: Gukov-Witten operators T_a $\quad$ a: conjugacy class of G
  - Charged: Wilson lines W_ρ $\quad$ ρ: irrep of G

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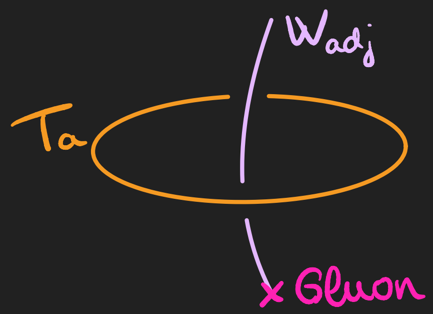
$$\boxed{B_\rho([a]) = \frac{\chi_\rho([a])}{\dim \rho}\, sz([a])}$$

- Strategy
  1. Write down all GW operators → connected component / → disconnected component
  2. Topological nature from trivial linking with W_{adj}

1-FORM SYMMETRY (CONT.)

- Trivial linking: $\chi_{adj}(a) = \dim \rho$
- Suppose T_a is topological



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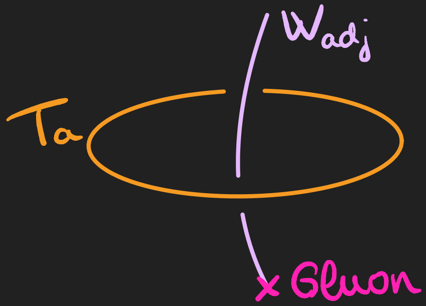

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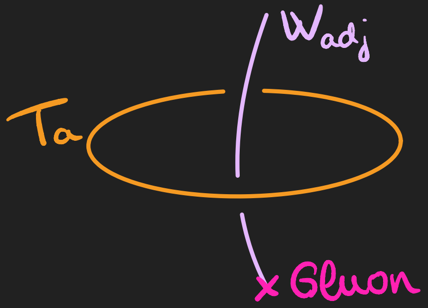

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- Add adjoint scalar and move in the Coulomb Branch

$G = U(1)^{\text{rank}} \rightarrow$ known answer: GW with trivial linking

Move back to the origin: $\{GW: \text{trivial linking}\} \subseteq \{GW: \text{top.}\}$

THE CASE OF PRINCIPAL EXTENSIONS

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- Fusion rule $\boxed{T_k^{\widetilde{SU}} \cdot T_q^{\widetilde{SU}} = T_{k+q}^{\widetilde{SU}} + T_{k-q}^{\widetilde{SU}}}$
- Argument doesn't depend on spacetime dimension

(d-2)- FORM SYMMETRY AND ALICE STRINGS

- Dual to the 1-form symmetry:
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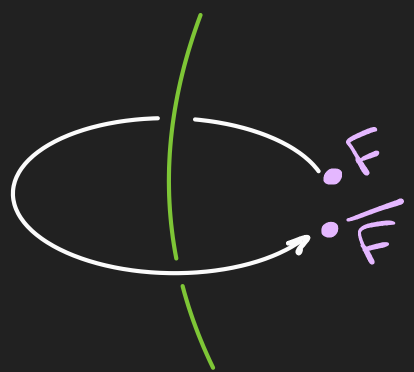
- (d-2)- form symmetry is broken if we add Alice strings



$\longrightarrow$ Action with the element

$$(1,-1) \in \widetilde{SU}(N)$$

BRANE CONSTRUCTIONS

- Branes intersecting orientifolds e.g.

(Harvey, Royston - '07)

	0	1	2	3	4	5	6	7	8	9
N D3	×	×	×	×						
O7 + D7	×	×			×	×	×	×	×	×

$\longrightarrow$ SQCD-like with a defect

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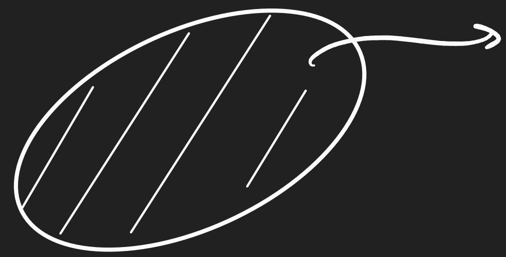
→ SQCD-like with a defect

- In 2d (x^0, x^1) $\widetilde{SU}$ gauge group

- In the worldvolume of the D3-branes:
  - The O-plane looks like an Alice string
  - What is the gauge group? ¿ $\widetilde{SU}$, Sp, SO ?

THE GAUGE GROUP AND THE ALICE STRING

- Alice strings reduce the globally well defined gauge group



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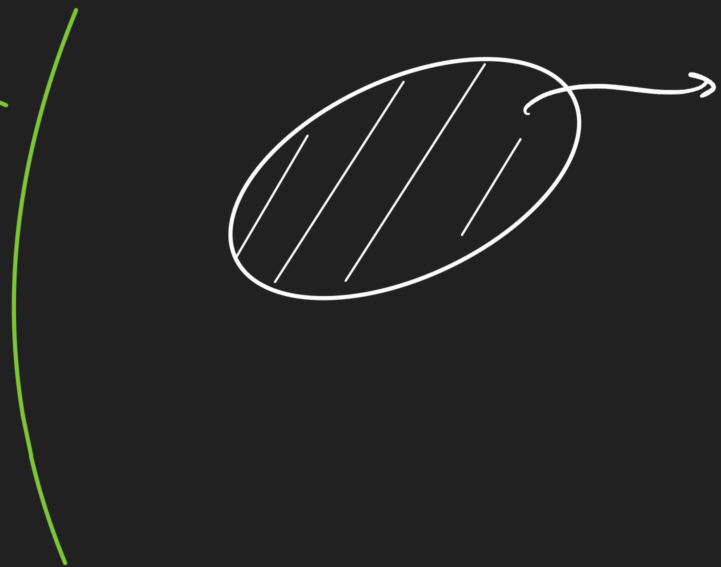
Add an Alice string



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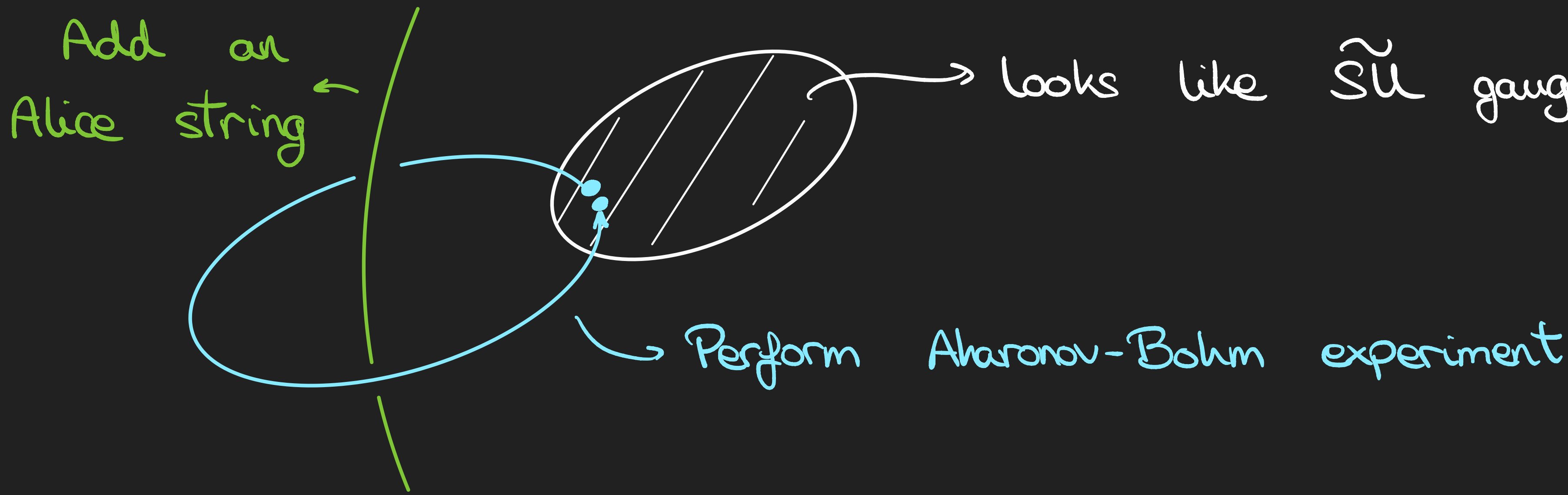

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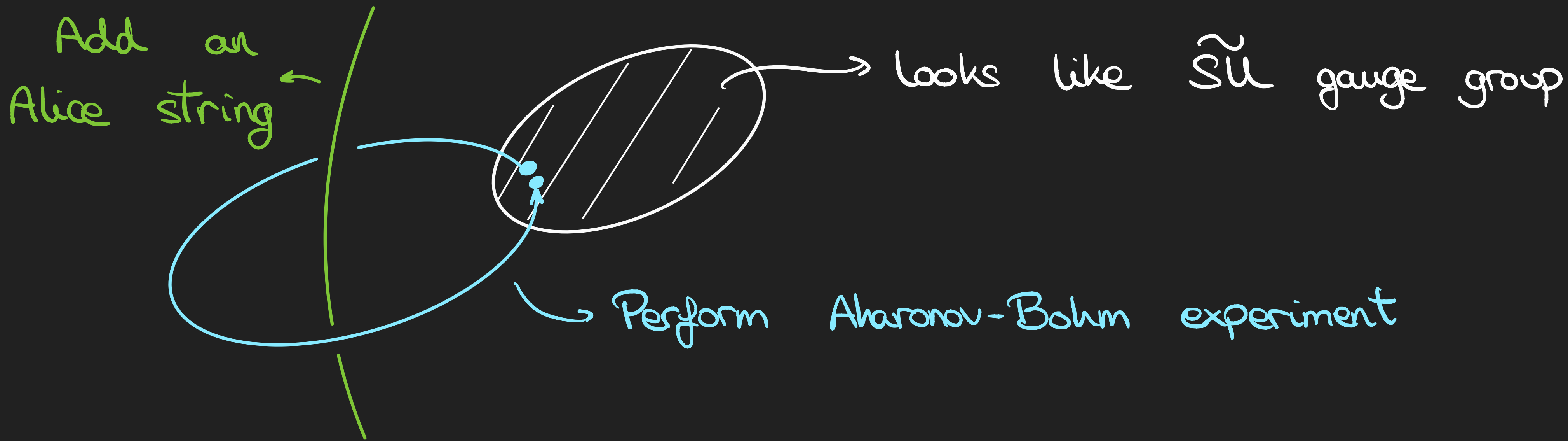

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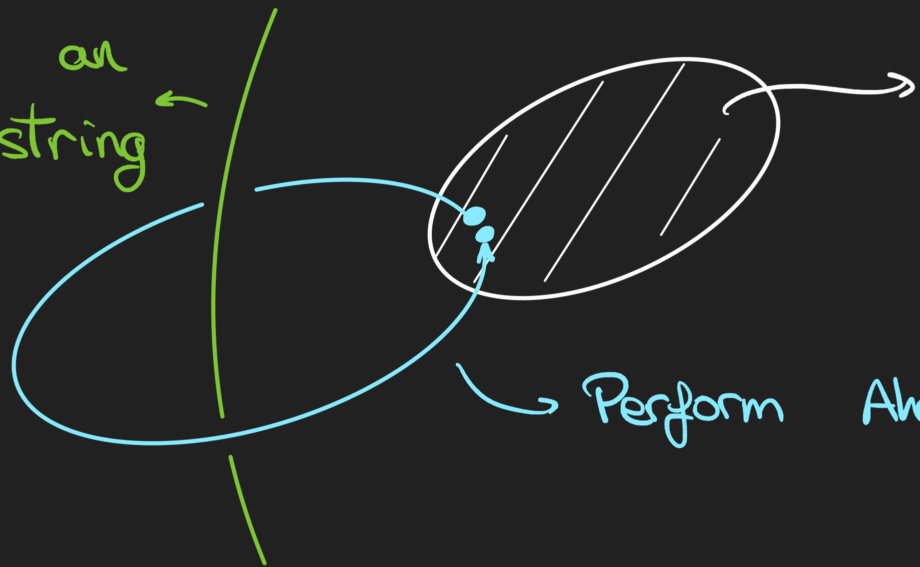

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¡CONTRADICTION!

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- Choice of semidirect product $\leftrightarrow$ type of orientifold

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- Gukov-Witten operators labelled by conjugacy classes of the gauge group
  - Invertible defects remain (but are further broken)
  - Non-invertible defects disappear

CONCLUSIONS AND OUTLOOK

- Pure $\widetilde{SU}(N)_{1,\ldots}$ has non-invertible 1-form symmetry

  (see also Laksha's talk)

- Brane constructions (preserving 8 supercharges) automatically break the higher-form symmetries

- Reminiscent of "No Global Symmetries" except for the fact that gravity is decoupled

- Can one find a stringy construction without the Alice string? ft. full $\widetilde{SU}(N)$ & non-invertible symmetries

THANK YOU FOR

YOUR ATTENTION !